11

The Fifth Quark

Discovery of the Υ and the B meson, 1977–1987

The discovery of the J/ψ and charmed quark seemed to complete a family of fermions, (c, s, ν_μ, μ), entirely analogous to (u, d, ν_e, e). If this pattern were indicative, then the τ and its neutrino presaged a new pair of quarks. Both e^+e^- annihilation and hadronic production of lepton pairs, the techniques that had uncovered the charmed quark, were extended in the search for the next quark.

Leon Lederman and his co-workers **(Ref. 11.1)** pressed the search for peaks in the $\mu^+\mu^-$ spectrum to high energies by studying the collisions of 400 GeV protons on nuclear targets at Fermilab. Their apparatus was a double-arm spectrometer set to measure $\mu^+\mu^-$ pairs with invariant masses above 5 GeV with a resolution of 2%. Hadrons were eliminated by using long beryllium filters in each arm. In mid 1977, a clear, statistically significant $\mu^+\mu^-$ peak was observed in the 9.5 GeV region with an observed width of about 1.2 GeV. A more detailed analysis showed better agreement with two peaks at 9.44 and 10.17 GeV, respectively, which were given the names Υ and Υ'. It soon became evident that this was a repetition of the J/ψ and ψ' story.

With the help of an energy upgrade, in May 1978 two groups at the DORIS e^+e^- storage ring at DESY were able to observe the Υ in the PLUTO and DASP II detectors. The results of the experiments are reproduced here (ref **11.2**, 11.3) and in Figure 11.47. The determination of the mass of the resonance was greatly improved with the result $M_\Upsilon = 9.46 \pm 0.01$ GeV. Moreover, the observed width was limited only by the energy spread of the beams, so that it was less than 1/100 as much as that observed in hadronic production. Just as for the J/ψ, it was possible to infer the partial width for $\Upsilon \to e^+e^-$ from the area under the resonance curve, with the result $\Gamma_{e^+e^-}(\Upsilon) = 1.3 \pm 0.4$ keV. Using nonrelativistic potential models derived from the ψ system and the assumption that the potential was independent of the quark type, it was possible to predict the wave function at the origin and thus $\Gamma_{e^+e^-}(\Upsilon)$ for the cases of charge $-1/3$ and $+2/3$. The comparison indicated that

Figure 11.47: Measurements of the e^+e^- cross section at the lower Υ states. Measurements from the DASP II experiment at DORIS show (a) the Υ and (b) the Υ' (Ref. 11.3). Measurements by the CLEO group at CESR show (c) the Υ, (d) the Υ', and (e) the Υ''. Discrepancies between the mass measurements by the two groups were later resolved (Ref. 11.5).

the new quark had charge $-1/3$ rather than $+2/3$. The new quark was dubbed the b for "bottom," reflecting the practice of writing the quark pairs (u, d) and (c, s) with the charge $-1/3$ beneath the charge 2/3 quark. Thus the prospective sixth quark is referred to as t or "top".

After additional cavities were added to increase the energy of the DORIS ring, the DESY–Heidelberg sodium iodide and lead glass detector and DASP II were able to observe the Υ' (Refs. 11.3, **11.4**). The $\Upsilon' - \Upsilon$ splitting was found to be very nearly the same as that for $\psi' - \psi$.

By 1980, the Cornell Electron Storage Ring (CESR) with its two detectors, CLEO and CUSB, became operational. They both observed the Υ and Υ', and additional resonances, Υ'' and Υ''' (Refs. 11.5, 11.6, 11.7, **11.8**). The first three states, with masses 9.460, 10.023, and 10.355 GeV, are narrow, with observed widths consistent with the beam spread of the machine. They are analogous to the ψ and ψ', and correspond to $1^3S_1, 2^3S_1$, and 3^3S_1 states of a $b\overline{b}$ system. Figure 11.47 shows the Υ and Υ' as observed by DASP II and the Υ, Υ', and Υ'' as observed by CLEO. The Υ''' at 10.577 GeV is a broader state, like the $\psi(3772) = \psi''$, and is interpreted as the 4^3S_1 state, lying above the threshold for $B\overline{B}$ production, where B represents a meson containing a $\overline{b}$ quark and a u or d quark. Thus $B^+ = \overline{b}u, B^0 = \overline{b}d, B^- = b\overline{u}, \overline{B}^0 = b\overline{d}$.

The existence of a series of s-wave bound states required that there be p-wave states as well. These were observed through radiative transitions from the s-wave states, $\Upsilon' \to \chi_b\gamma$, where χ_b represents a $C = +1$, $P = +1$ p-wave state. Evidence was obtained from the inclusive photon spectrum, $\Upsilon' \to \gamma +$ anything, and from the cascade $\Upsilon' \to \gamma\chi_b$, $\chi_b \to \gamma\Upsilon$, $\Upsilon \to l^+l^-$, where l represents e or μ. Measurements were carried out by CUSB and CLEO at CESR and by the Crystal Ball at DORIS II after the detector was shipped from Stanford to Hamburg. In Figure 11.48 some results from the Crystal Ball are shown.

What is the role of the b quark in weak interactions? Beta decay is described at the quark level by the transition $d \to ue^-\overline{\nu}$. Positron emission is the result of $u \to de^+\nu$. The strangeness-changing semileptonic weak decays (e.g. $\Lambda \to pe\overline{\nu}$) are described by $s \to ue^-\overline{\nu}$ whose inverse is $u \to se^+\nu$. The Cabibbo hypothesis is that the weak current is really $u \to (\cos\theta_c d + \sin\theta_c s)$. As discussed in Chapter 9, the introduction of a fourth quark makes the Cabibbo angle into a rotation, with the current described by

$$\begin{pmatrix} \overline{u} & \overline{c} \end{pmatrix} \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

The V-A structure $\gamma_\mu(1-\gamma_5)$ has been suppressed for clarity. The 2×2 matrix can be viewed either as a rotation of the charge $-1/3$ quarks or of the charge $+2/3$ quarks, though by convention it is usually the charge $-1/3$ quarks that are subjected to rotation.

With the discovery of the b quark it was apparent that the Cabibbo matrix

Figure 11.48: The photon spectrum from Υ' decays obtained by the Crystal Ball Collaboration at DORIS II. A triplet of lines corresponding to $\Upsilon' \to \gamma\chi_b(^3P_{2,1,0})$ is seen between 100 and 200 MeV. The decays $\chi_b \to \gamma\Upsilon$ produce the unresolved signal between 400 and 500 MeV [R. Nernst *et al.*, *Phys. Rev. Lett.* **54**, 2195 (1985)].

would have to be expanded to a 3×3 matrix. Indeed this possibility had been anticipated by M. Kobayashi and T. Maskawa before the discovery of even the charmed quark. They observed that if there were a third generation, that is a third pair like (u, d) and (c, s), the 3×3 mixing matrix would allow for CP violation.

In order to provide for CP violation, we need a complex term in the interaction $J_\mu^\dagger J^\mu$ where $J_\mu = \overline{U}\gamma_\mu V(1-\gamma_5)D$ is the weak current. If there are n families, U represents the column of n charge $2/3$ quarks and D the column of n charge $-1/3$ quarks. The matrix V is unitary and has n^2 complex or $2n^2$ real parameters. Unitarity imposes the conditions $V_{ij}V_{kj}^* = \delta_{ik}$, which give $n(n-1)/2$ complex constraints for $i \neq k$ and n real constraints for $i = k$. Altogether there are n^2 remaining free parameters in V.

It is possible to eliminate some of the complex phases in V by redefining the phases of the $2n$ quark fields. Changing all of the fields by the same phase changes nothing so $2n-1$ phases from V can be eliminated in this way. Thus the number of real parameters characterizing V is $n^2 - 2n + 1 = (n-1)^2$. For two families this gives just one parameter, which is the Cabibbo angle. For three families there are 4 parameters. Now if V were purely real it would be a 3×3 rotation matrix, which is determined by three real parameters. Thus the fourth parameter of V must necessarily introduce a complex component into V, one that cannot be absorbed into a redefinition of the quark fields.

We can represent the Kobayashi–Maskawa (KM) matrix by

$$\begin{bmatrix} \overline{u} & \overline{c} & \overline{t} \end{bmatrix} \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

In principle, the squares of the various matrix elements can be measured by observing a variety of weak decays. The comparison of nuclear beta decay and muon decay indicates $|V_{ud}| \approx 0.97$, while the strangeness-changing decays give $|V_{us}| \approx 0.22$. These two are just $\cos\theta_c$ and $\sin\theta_c$ in the Cabibbo scheme. The production of charmed particles in neutrino (or antineutrino) nucleus scattering is proportional to $|V_{cd}|^2$. Data from the CDHS Collaboration led to a value $|V_{cd}| = 0.21 \pm 0.03$. The decay of mesons containing b quarks is controlled by V_{ub} and V_{cb}. The relative size of these elements determines the ratio of the semileptonic decays $\Gamma(b \to ul\nu)/\Gamma(b \to cl\nu)$. Because of the greater phase space available, the $b \to ul\nu$ decay produces leptons with higher momentum than does $b \to cl\nu$. In Figure 11.49 we show data for the lepton spectra. The evidence overwhelmingly supports $b \to cl\nu$ as the dominant mode. Recent data indicate that

$$\frac{\Gamma(b \to ul\nu)}{\Gamma(b \to cl\nu)} < 0.08$$

Correcting for the difference in phase space available for the two modes gives

$$|V_{ub}/V_{cb}| < 0.22$$

Actual identification of B meson decays promised to be a formidable task, even though some lessons had been learned from the study of charm. By focusing on the 4^3S_1 Υ''' it was possible to obtain a good sample of $\Upsilon''' \to B\overline{B}$ events **(Ref. 11.9)**. The technique used was to identify candidates for D^0 s and D^{*+} s, using only entirely charged decay modes, and combine these with either one or two charged pions. In analogy with the fundamental decay $c \to s$ leading to $D^+ \to K^-\pi^+\pi^+$, the transition $b \to c$ produces $B^- \to D^+\pi^-\pi^-$, $D^{*+}\pi^-\pi^-$. The combinations $B \to D\pi$ and $B \to D\pi\pi$ were required to produce Bs with energy equal to the beam energy since the decay is $\Upsilon''' \to B\overline{B}$. An accumulation of events for mass near 5.275 GeV demonstrated the observation of exclusive B decays.

The CLEO Collaboration at CESR and the ARGUS Collaboration at DESY have identified additional decay modes. Particularly noteworthy are $B^+ \to \psi K^+$ and $B^+ \to \psi' K^+$.

The CUSB Collaboration has observed photons of energy about 50 MeV associated with $B\overline{B}$ production at energies above the Υ'''. This is ascribed to the production of $B^*\overline{B}$ and the subsequent decay $B^* \to B\gamma$. The splitting between the spin-one B^* and the pseudoscalar B was determined to be $52 \pm 2 \pm 4$ MeV. Some of the CUSB data are shown in Figure 11.50.

Figure 11.49: Lepton spectra for semileptonic B meson decays. On the left, CUSB data from CESR together with the curves expected for (A) $b \to ce\nu$, (B) $b \to ue\nu$, and (C)$b \to cX$, $c \to se\nu$ [C. Klopfenstein *et al.*, *Phys. Lett.* **130B**, 444 (1983)]. On the right, data from CLEO, also taken at CESR. The upper figure is for electrons and the lower for muons. The solid curves are predictions without any $b \to ul\nu$ while the dotted curves are predictions for purely $b \to ul\nu$ [A. Chen *et al.*, *Phys. Rev. Lett.* **52**, 1084, (1984)]. All the figures indicate that $\Gamma(b \to cl\nu) >> \Gamma(b \to ul\nu)$. This Kobayashi-Maskawa suppression is analogous to the Cabibbo suppression observed in the decays of charmed particles to states without strangeness.

Figure 11.50: The photon energy spectrum obtained by the CUSB Collaboration for events with high energy leptons and thrust less than 0.88 (indicative of events more spherical than the ordinary two-jet events produced in e^+e^- annihilation). These criteria signal the presence of B mesons. The e^+e^- c.m. energy for the solid histogram in (a) is 10.62 – 11.25 GeV, above the $B^*\overline{B}$ threshold. The dotted histogram in (a) was taken at the Υ''', below the $B^*\overline{B}$ threshold. In (b) the spectrum with the background subtracted shows a line near 50 MeV, ascribed to $B^* \to B\gamma$ (K. Han *et al.*, *Phys. Rev. Lett.* **55**, 36 (1985)).

Semileptonic decays have also been exploited in several experiments to obtain B-enriched samples of events for B lifetime measurements. The Mark II Collaboration at PEP built a vertex detector using a precision drift chamber located close to the interaction point, which allowed measurements of the distance of closest approach of the lepton tracks to the beam–beam collision region. The experiment found a B lifetime of about 1 ps. Similar measurements were made by the MAC, TASSO, DELCO, and JADE experiment at PEP and PETRA. This unexpectedly long lifetime indicated that the V_{cb} matrix element is quite small, a current estimate being between 0.030 and 0.062. From the unitarity of the KM matrix, we conclude that $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$, so $|V_{cs}| \approx 0.97$, assuming there are just three generations.

Just as for K^0 and $\overline{K}^0$, there can be mixing between the neutral B^0 and $\overline{B}^0$. In fact, this is possible for two distinct systems, the non-strange $B_d^0 = \overline{b}d$ and the strange $B_s^0 = \overline{b}s$. The mixing does not require CP violation, but depends only on the existence of common states to which both B^0 and $\overline{B}^0$ can decay. The B^0 favors decays to states like $\overline{D}\pi$, while the $\overline{B}^0$ prefers $D\pi$. Both however, can decay to $D\overline{D}$, albeit as a Cabibbo suppressed decay. Similarly, they both can decay virtually to states like $T\overline{T}$, where T represents the as yet undiscovered pseudoscalar meson made from $t\overline{u}$ or $t\overline{d}$. Of course, multiparticle states are possible as well.

Unlike the K^0, the B^0 has many decay channels open to it with $CP = -1$ and with $CP = +1$. As a result, the two CP eigenstates made from B^0 and $\overline{B}^0$ must have very similar lifetimes. The mixing parameter, r, introduced in Chapter 7 thus becomes

$$r = \frac{(\Delta m/\Gamma)^2}{2 + (\Delta m/\Gamma)^2}$$

An analogous equation holds for $D^0 - \overline{D}^0$ mixing. Theoretical predictions for mixing are based on processes shown in Figure 15.57. Mixing depends on both the quark masses and the Kobayashi-Maskawa matrix. If the d, s, and b quarks were degenerate in mass, we could redefine them so that the u quark decayed entirely to d, c entirely to s, and t entirely to b. Then the Kobayashi-Maskawa matrix would be the unit matrix and there would be no intermediate quark states possible in Figure 15.57. The effect, then, depends critically on quark mass differences, emphasizing the importance of the heavy quarks.

If a $B\overline{B}$ pair is created and both decay semileptonically, the B would be expected to give a positive lepton ($\overline{b} \to \overline{c}l^+\nu$) and the $\overline{B}$ a negative lepton. If there is B–$\overline{B}$ mixing, it is possible that both leptons will have the same sign. An unfortunate background arises from the chain $b \to c \to Xl\nu$ since the semileptonic decay of the c would give a lepton of the sign opposite that expected from a b decay. While some evidence for $B - \overline{B}$ mixing was found by UA-1 at the Sp$\overline{\text{p}}$S in the same-sign dilepton signal, clear convincing evidence was first obtained in an e^+e^- experiment.

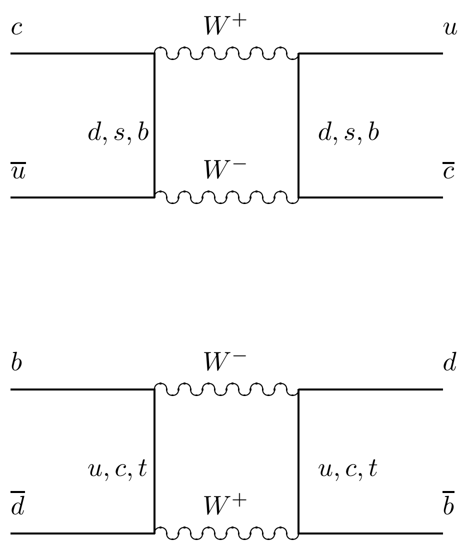

Figure 11.51: The diagrams contributing to mixing of D^0 with $\overline{D}^0$ and B^0 with $\overline{B}^0$. The relative strength of different contributions depends on the Kobayashi-Maskawa matrix and the quark masses. For the D^0, the s quark intermediate is most important. Precise predictions cannot be made, but $D^0 - \overline{D}^0$ mixing is expected to be small and has not been observed. For $B^0 - \overline{B}^0$ mixing is dominated by the t quark intermediate state.

Searches by CLEO at the $\Upsilon'''$and by Mark II at 29 GeV produced upper limits for $B - \overline{B}$ mixing expressed in terms of

$$r_d = \Gamma(B_d^0 \to e^- X)/\Gamma(B_d^0 \to e^+ X)$$

$$r_s = \Gamma(B_s^0 \to e^- X)/\Gamma(B_s^0 \to e^+ X)$$

The CLEO result $r_d < 0.24$ (at 90% confidence level) was restricted to B_d since the data were taken below the B_s threshold (Ref. 11.10), while the Mark II limit was for a combination of r_d and r_s (Ref. 11.11). The UA-1 experiment measured a value of $r = 0.42 \pm 0.07$ with an expected background of 0.26 ± 0.03. The difference they interpreted as being due to $B_s - \overline{B}_s$ mixing, which was expected to be much larger than $B_d - \overline{B}_d$ mixing (Ref. 11.12).

The ARGUS Collaboration finally provided convincing evidence for $B_d - \overline{B}_d$ mixing with studies at the Υ'''(**Ref. 11.13**). One event was found where $\Upsilon''' \to B_d^0 B_d^0$ as demonstrated by specific semileptonic decay modes, each with a positive muon. Additional evidence for mixing was obtained by measuring the like-sign dilepton signal. A third independent measurement came from identifying complete B^0 decays and observing semileptonic decays of the accompanying $\overline{B}^0$. Finding a positive lepton opposite an identified B^0 is evidence for mixing. Combining the results of these measurements gave $r_d = 0.21 \pm 0.08$. In view of this result, the UA-1 same-sign dileptons are probably due to both B_d^0 and B_s^0 mixing.

The discovery of the b quark provided an excellent opportunity to test the models proposed to explain the phenomena associated with the charmed quark. These tests have been quite successful in a qualitative and semiquantitative way. The general spacing of bound-state levels in the two systems can be understood from a single potential. The systematics of the fine structure (the splitting of the p-wave states) is in accord with expectations. The rates for radiative decays are in general agreement with the nonrelativistic model. The mixing of the B_d^0 and $\overline{B}_d^0$ was a beautiful reprise of the K^0–$\overline{K}^0$ mixing first observed thirty years before. The b quark provided, as well, a possible explanation for CP violation through the complex phase in the Kobayashi–Maskawa matrix. A test of this mechanism awaits further experimental and theoretical progress. It awaits, as well, the discovery of the presumed t quark.

EXERCISES

11.1 The Wolfenstein parameterization of the Kobayashi-Maskawa matrix is

$$V = \begin{bmatrix} 1-\lambda^2/2 & \lambda & \lambda^3 A(\rho - i\eta) \\ -\lambda & 1-\lambda^2 & \lambda^2 A \\ \lambda^3 A(1-\rho-i\eta) & -\lambda^2 & 1 \end{bmatrix}$$

where λ, ρ, η, and A are real parameters. Show that this matrix is unitary ($VV^\dagger = I$) up to corrections of order λ^4. Take $V_{us} = 0.22$, $|V_{cb}|$=0.045, and $|V_{ub}/V_{cb}|$= 0.20. Find λ, A, and $\rho^2+\eta^2$. See L. Wolfenstein, *Phys. Rev. Lett.* **51**, 1945 (1983).

11.2 Show that if a $B^0\overline{B}^0$ pair is produced in e^+e^- annihilation in association with other particles far above the $B\overline{B}$ threshold, if both Bs decay semileptonically, the like to unlike sign ratio is

$$\frac{N(l^+l^+) + N(l^-l^-)}{N(l^+l^-)} = \frac{2r}{1+r^2}$$

but if the pair is produced by the $\Upsilon'''(4^3S_1)$ the ratio is simply r.

11.3 Suppose the quark–antiquark potential obeys the power law $V(r) = ar^\nu$. Show that the binding energies vary with the quark mass as $E \propto m^{-\nu/(\nu+2)}$ and that the density at the origin $|\phi(0)|^2$ for an s-wave state varies as $m^{3/(\nu+2)}$. Given that the splitting $\Upsilon' - \Upsilon$ is nearly identical to that for $\psi' - \psi$, predict $\Gamma(\Upsilon \to e^+e^-)$ from $\Gamma(\psi \to e^+e^-)$ if the charge of the new quark is $-1/3$ or is $+2/3$.

11.4 Show that $|\phi(0)|^2$, the s-wave wave function at the origin squared, is related to the average force by

$$|\phi(0)|^2 = \frac{m}{2\pi} < F >$$

Hint: write the Schrödinger wave equation for the radial wave function, u and multiply by u'. Integrate the result from $r = 0$ to $r = \infty$.

11.5 Use the results of Exercise 6.5 to determine the e^- spectrum in the decays $b \to ce^-\overline{\nu}$, $b \to ue^-\overline{\nu}$. Take $m_b = 5$ GeV, m_c= 1.5 GeV, m_u= 0.3 GeV.

11.6 * Suppose that the $b\overline{b}$ or $c\overline{c}$ interaction can be represented approximately by a nonrelativistic Schrödinger equation:

$$\left[-\frac{1}{2m_{red}}\nabla^2 + V(r)\right]\psi = E\psi$$

where $m_{red} = m/2$ is the reduced mass. Then the energy levels are spin-independent so ${}^3S_1$ and ${}^1S_0$ are degenerate, as are ${}^3P_{2,1,0}$ and ${}^1P_1$, etc. Now consider as perturbations the spin dependent forces

$$\begin{aligned}
&\mathbf{L}\cdot\mathbf{S}\, V_{so}(r) \\
&\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2}\, V_{spin-spin}(r) \\
&\frac{3\,\boldsymbol{\sigma_1}\cdot\mathbf{r}\,\boldsymbol{\sigma_2}\cdot\mathbf{r} - \boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2} r^2}{r^2} V_{tensor}(r) \equiv S_{12} V_{tensor}(r)
\end{aligned}$$

Here $\mathbf{S} = \frac{1}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)$ is the total spin, and $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the quark and antiquark spin operators.

a. Which degenerate states are split by each of the interactions? Which nondegenerate state are mixed by the interactions?

b. Use the relations (try to prove them, too)

$$\begin{aligned}
&< J, L = J+1, \quad S = 1, M|S_{12}|J, L = J+1, \quad S = 1, M >= -2\frac{L+1}{2L-1} \\
&< J, L = J, \quad S = 1, M|S_{12}|J, L = J, \quad S = 1, M >= +2 \\
&< J, L = J-1, \quad S = 1, M|S_{12}|J, L = J-1, \quad S = 1, M >= -2\frac{L}{2L+3}
\end{aligned}$$

to analyze the observed splittings of the ${}^3P$ states in the Υ and ψ systems. Here $|J, L, S, M >$ is a state with total angular momentum J, orbital angular momentum L, total spin angular momentum S, and $J_Z = M$. [See J. D. Jackson, “Lectures on the New Particles” in *Proc. of Summer Institute on Particle Physics, Stanford, CA, Aug. 2-13, 1976*, M. Zipf, ed.]

11.7 * The relation between the standard relativistic Lorentz invariant amplitude, $\mathcal{M}$, (the usual Feynman rules generate $-i\mathcal{M}$) and the conventional scattering amplitude of potential theory is

$$f = -\frac{1}{8\pi\sqrt{s}}\mathcal{M}$$

where s is the square of the center of mass energy. The center-of-mass differential cross section is $d\sigma/d\Omega = |f|^2$. In potential theory, the Born value for f is

$$f = -\frac{m}{2\pi}\int d^3r e^{-i\mathbf{p}'\cdot\mathbf{r}}V(r)e^{i\mathbf{p}\cdot\mathbf{r}}$$

where $\mathbf{p}$ and $\mathbf{p}'$ are the initial and final momenta. Two body scattering can be treated analogously with the modification $m \to m_{reduced}$. If the particles have spin 1/2, we generalize the wave function to $\psi(r)\chi_1\chi_2$, where χ_1 and χ_2 are two-component spinors. Thus

$$f = -\frac{m_{red}}{2\pi}\int d^3r\chi_2'^\dagger\chi_1'^\dagger e^{-i\mathbf{p}'\cdot\mathbf{r}}V(r)e^{i\mathbf{p}\cdot\mathbf{r}}\chi_1\chi_2$$

Suppose $\mathcal{M}$ has the form of vector exchange but with some more general dependence on momentum transfer:

$$\mathcal{M} = \overline{u}(p_4)\gamma_\mu u(p_2)\ \overline{u}(p_3)\gamma^\mu u(p_1)\ \tilde{V}(p_1 - p_3)$$

Show that the spin dependent potential is, to leading order

$$V(r) = V_0(r) + \frac{3}{2m^2}\frac{1}{r}\frac{dV_0}{dr}\mathbf{L}\cdot\mathbf{S} + \frac{1}{12m^2}S_{12}\left(\frac{1}{r}\frac{dV_0}{dr} - \frac{d^2V_0}{dr^2}\right) + \frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{6m^2}\nabla^2V_0$$

where $\mathbf{S} = \frac{1}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)$ is the total spin and

$$\int d^3r\, V_0(r)e^{i\mathbf{q}\cdot\mathbf{r}} = \tilde{V}(q)$$

BIBLIOGRAPHY

For a discussion of the Kobayashi–Maskawa matrix see the article in the 1988 edition of the *Review of Particle Properties* by F. J. Gilman K. Kleinknecht, and B. Renk.

For a review of Υ physics, see P. Franzini and J. Lee-Franzini, *Ann. Rev. Nucl. Part. Sci.* **33**, 1 (1983).

B meson decays are reviewed in E. H. Thorndike and R. A. Poling, *Phys. Rep.*, **157**, 183 (1987).

REFERENCES

11.1 S. W. Herb *et al.*, "Observation of a Dimuon Resonance at 9.5 GeV in 400 GeV Proton Nucleus Collisions." *Phys. Rev. Lett.*, **39**, 252 (1977).

11.2 PLUTO Collaboration, Ch. Berger *et al.*, "Observation of a Narrow Resonance formed in e^+e^- Annihilation at 9.46 GeV." *Phys. Lett.*, **76B**, 243 (1978).

11.3 C. W. Darden *et al.*, "Observation of a Narrow Resonance at 9.46 GeV in Electron-Positron Annihilation." *Phys. Lett.*, **76B**, 246 (1978). Also *ibid.* **78B**, 364 (1978).

11.4 J. K. Bienlein *et al.*, "Observation of a Narrow Resonance at 10.02 GeV in e^+e^- Annihilation." *Phys. Lett.*, **78B**, 360 (1978).

11.5 D. Andrews *et al.*, "Observation of Three Upsilon States." *Phys. Rev. Lett.*, **44**, 1108 (1980).

11.6 T. Böhringer *et al.*, "Observation of Υ, Υ', and Υ'' at the Cornell Electron Storage Ring." *Phys. Rev. Lett.*, **44**, 1111 (1980).

11.7 D. Andrews *et al.*, "Observation of a Fourth Upsilon State in e^+e^- Annihilation." *Phys. Rev. Lett.*, **45**, 219 (1980).

11.8 G. Finocchiaro *et al.*, "Observation of the Υ''' at the Cornell Electron Storage Ring." *Phys. Rev. Lett.*, **45**, 222 (1980).

11.9 S. Behrends *et al.*, "Observation of Exclusive Decay Modes of b Flavored Mesons." *Phys. Rev. Lett.*, **50**, 881 (1983).

11.10 A. Bean *et al.*, "Limits on $B^0\overline{B}^0$ Mixing and τ_{B^0}/τ_{B^+}." *Phys. Rev. Lett.*, **58**, 183 (1987).

11.11 T. Schaad *et al.*, "Upper Limit on $B^0\overline{B}^0$ Mixing in e^+e^- Annihilation at 29 GeV." *Phys. Lett.*, **160B**, 188 (1985).

11.12 C. Albajar *et al.*, "Search for $B^0\overline{B}^0$ Oscillations at the CERN Proton-Antiproton Collider." *Phys. Lett.*, **186B**, 247 (1987).

11.13 H. Albrecht *et al.*, "Observation of $B^0\overline{B}^0$ Mixing." *Phys. Lett.*, **192B**, 245 (1987).