7

The neutral kaon system

From the discovery of the K_L^0 to CP violation, 1956–1967

The development of the concept of strangeness created something of a puzzle: What is the nature of the K^0 and $\overline{K}^0$? They differ only in their strangeness, a quantity not conserved by the weak interactions through which they decay. Thus, for example, they both can decay into $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$. The explanation was given by Gell-Mann and Pais before parity violation was discovered. We present their proposal modified to incorporate parity violation, but assuming at first that the combination, CP, of charge conjugation and parity inversion is a good symmetry of both the weak and strong interactions.

The K^0 is an eigenstate of the strong interactions, as is the $\overline{K}^0$. They are antiparticles of each other so they can be transformed into each other by charge conjugation and thus have opposite strangeness. If there were no weak interactions, the K^0 and $\overline{K}^0$ would be degenerate, that is, equal in mass. The weak interactions break the degeneracy and make the neutral kaons unstable. The particles with well-defined masses and lifetimes are the physical states, the eigenstates of the total Hamiltonian, including both strong and weak interactions. These states are linear combinations of K^0 and $\overline{K}^0$, the strong interaction eigenstates.

Since the action of CP on a K^0 produces a $\overline{K}^0$ we can establish a phase convention by

$$CP|K^0\rangle = -|\overline{K}^0\rangle$$

If CP is conserved, the physical eigenstates are the eigenstates of CP. These are simply

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle - |\overline{K}^0\rangle\right]$$

$$|K_2^0\rangle = \frac{1}{\sqrt{2}}\left[|K^0\rangle + |\overline{K}^0\rangle\right]$$

where K_1^0 has $CP = +1$ and K_2^0 has $CP = -1$. The decays $K^0 \to \pi^+\pi^-$ and $\overline{K}^0 \to \pi^+\pi^-$ are both allowed by the weak-interaction selection rules. The $\pi^+\pi^-$ state with angular momentum 0 necessarily has $P = (-1)^L = +1$, $C = (-1)^L = +1$ since both C and P interchange the two pions, which are in an s-wave, and thus $CP = +1$. It follows that the K_2^0 cannot decay into $\pi^+\pi^-$ if CP is conserved. On the other hand, a $\pi^+\pi^-\pi^0$ state that is entirely s-wave must have $CP = -1$ because the $\pi^+\pi^-$ part has $CP = +1$ by the above reasoning, while the remaining π^0 has $CP = -1$. Since the important decay channel $\pi\pi$ is closed to it, the K_2^0 has a longer lifetime than the K_1^0.

Because K^0 and $\overline{K}^0$ are the strong interaction eigenstates, in hadronic collisions it is these that are directly produced. According to Gell-Mann and Pais, a produced K^0 is to be regarded as a superposition of a CP–even K_1^0 and an CP–odd K_2^0. The K_1^0 portion of the state decays much more rapidly that the K_2^0 portion, so that after a period of time only the latter is present if the particle has not yet decayed. While decays into $\pi\pi$ or $\pi\pi\pi$ are possible from either K^0 or $\overline{K}^0$, by the $\Delta S = \Delta Q$ rule, a decay to $e^+\nu\pi^-$ is possible only from K^0, while a decay to $e^-\nu\pi^+$ must come from $\overline{K}^0$.

The K_2^0 was observed in 1956 by Lande *et al.* using a 3-GeV beam from the Brookhaven Cosmotron (**Ref. 7.1**). A cloud chamber filled 90% with helium and 10% with argon was placed 6 m from the interaction point. All K_1^0s and Λs would have decayed by the time of their arrival at the cloud chamber. In the cloud chamber, forked tracks were observed that were kinematically unlike $\theta^0 \to \pi^+\pi^-$. It was concluded that they represented $\pi^\pm e^\mp \nu$, possibly $\pi^\pm \mu^\mp \nu$, and occasionally $\pi^+\pi^-\pi^0$. The lifetime was judged to be in the range 10^{-9} s $< \tau < 10^{-6}$ s, whereas the short-lived $K^0\,(\theta)$ had a lifetime around 10^{-10} s. Additional evidence for a long-lived neutral K was obtained by W. F. Fry and co-workers using a K^- beam from the Bevatron with an emulsion target (**Ref. 7.2**).

These results were followed by a more complete report by Lande, Lederman, and Chinowsky showing clearly the $\mu\pi\nu$, $e\pi\nu$, and 3π modes (Ref. 7.3). They obtained further confirmation of the Gell-Mann–Pais prediction by noting a neutral K that interacted with a helium nucleus to produce $\Sigma^- ppn\pi^+$, a state with negative strangeness. The neutral K beam was overwhelmingly of positive strangeness initially since the threshold for $pn \to p\Lambda K^0$ is much lower than that for, say, $pn \to pnK^0\overline{K}^0$. Thus there was strong evidence for the transformation $K^0 \to \overline{K}^0$.

In vacuum, the time development of the K_1^0 and K_2^0 is

$$|K_1^0(\tau)\rangle = e^{-im_1\tau - \gamma_1\tau/2}\frac{1}{\sqrt{2}}\left[|K^0(0)\rangle - |\overline{K}^0(0)\rangle\right]$$

$$|K_2^0(\tau)\rangle = e^{-im_2\tau - \gamma_2\tau/2}\frac{1}{\sqrt{2}}\left[|K^0(0)\rangle + |\overline{K}^0(0)\rangle\right]$$

where $m_{1,2}$ and $\gamma_{1,2}$ are the masses and decay rates of the K_1^0 and K_2^0. Here τ is the proper time, $\tau = t(1 - v^2)^{1/2}$, t is the time measured in the laboratory, and as usual the speed of light, $c = 1$. Because of virtual weak transitions between the K^0 and $\overline{K}^0$, the masses m_1 and m_2 differ slightly. If a state, $|\Psi\rangle$, that is purely K^0 is produced at $\tau = 0$, it will oscillate between K^0 and $\overline{K}^0$ with amplitudes

$$\langle K^0|\Psi(\tau)\rangle = \frac{1}{2}(e^{-im_1\tau-\gamma_1\tau/2} + e^{-im_2\tau-\gamma_2\tau/2})$$
$$\langle \overline{K}^0|\Psi(\tau)\rangle = \frac{1}{2}(-e^{-im_1\tau-\gamma_1\tau/2} + e^{-im_2\tau-\gamma_2\tau/2})$$

These oscillations can be observed through semileptonic decays since, by the $\Delta S = \Delta Q$ rule, the semileptonic decays are $K^0 \to \pi^- e^+ \nu$ and $\overline{K}^0 \to \pi^+ e^- \overline{\nu}$. An example is shown in Figure 7.25. There the charge asymmetry in the decay of K^0 is shown as a function of the proper time. The ratio of "wrong sign" leptons (e^-) to "right sign" leptons (e^+) from a state that is initially a K^0, integrated over all time, is

$$r = \frac{(\gamma_1 - \gamma_2)^2 + 4(\Delta m)^2}{2(\gamma_1 + \gamma_2)^2 - (\gamma_1 - \gamma_2)^2 + 4(\Delta m)^2}$$

Since the decay rate of the K_1^0, γ_1, is much greater than γ_2, the K_2^0 decay rate, this ratio is nearly unity.

Even more dramatic predictions had been made for the neutral K system. Pais and Piccioni in 1955 predicted that K_2^0s passing through matter would regenerate a coherent K_1^0 component. In matter, the time development is altered because the K^0 and $\overline{K}^0$ interact differently with nucleons. For example, $\overline{K}^0 p \to \pi^+ \Lambda$ is allowed while $K^0 p \to \pi^+ \Lambda$ is not. In fact, the elastic scattering amplitudes, f and $\overline{f}$, for $K^0 p$ and $\overline{K}^0 p$ differ, just as those for $K^+ p \to K^+ p$ and $K^- p \to K^- p$ do. Forward-moving neutral kaons accumulate extra phase from elastic scattering. As in ordinary electromagnetic interactions, this scattering can be translated into an index of refraction

$$n = 1 + \frac{2\pi N}{k^2} f(0)$$

where N is the number density of scatterers, k is the wave number of the incident particles, and $f(0)$ is the (complex) elastic-scattering amplitude in the forward direction, which is related to the total cross section by the optical theorem

$$\sigma_{tot} = \frac{4\pi}{k} \text{Im} f(0)$$

Since K^0 and $\overline{K}^0$ have different total cross sections, they have different (complex) indices of refraction. In going a distance l, a particle picks up an extra phase $k(n-1)l$. The distance l is related to the proper time interval by $l = \tau v/(1-v^2)^{1/2}$.

To incorporate this effect, we write first the Schrödinger equation for propagation in free space. It is easy to guess what this is since we already have the

Figure 7.25: The charge asymmetry observed for $K^0 \to \pi^- e^+ \nu$ and $\overline{K}^0 \to \pi^+ e^- \overline{\nu}$ as a function of the proper time, starting from a predominantly K^0 beam. The number of observed positrons is N^+ and the number of observed electrons is N^-.The interference effect seen is sensitive to the $K_L - K_S$ mass difference. For large values of the proper time, the nonzero asymmetry is a CP violating effect and determines $\mathrm{Re}\,\epsilon$ [S. Gjesdal *et al.*, *Phys. Lett.* **52B**, 113 (1974)]. This CP violating effect was first observed in Refs. 7.6 and 7.7.

solutions in the form of $|K_1^0(\tau)\rangle$ and $|K_2^0(\tau)\rangle$. If we let ψ be a column matrix whose upper entry gives the K^0 component and whose lower entry gives the $\overline{K}^0$ component, then in terms of the proper time

$$i\frac{\partial\psi}{\partial\tau} = \begin{pmatrix} \overline{m} - i\dfrac{\overline{\gamma}}{2} & \delta m - i\dfrac{\delta\gamma}{2} \\ \delta m - i\dfrac{\delta\gamma}{2} & \overline{m} - i\dfrac{\overline{\gamma}}{2} \end{pmatrix} \psi$$

where $\overline{m} = (m_2+m_1)/2$, $\overline{\gamma} = (\gamma_2+\gamma_1)/2$, $\delta m = (m_2-m_1)/2$, and $\delta\gamma = (\gamma_2-\gamma_1)/2$. We indicate the elastic K^0-nucleus scattering amplitude by f and that for $\overline{K}^0$ by $\overline{f}$. With the inclusion of the effects of the medium we have

$$i\frac{\partial\psi}{\partial\tau}=\begin{pmatrix} \overline{m}-i\frac{\overline{\gamma}}{2}-\frac{2\pi Nvf}{k(1-v^2)^{1/2}} & \delta m-i\frac{\delta\gamma}{2} \\ \delta m-i\frac{\delta\gamma}{2} & \overline{m}-i\frac{\overline{\gamma}}{2}-\frac{2\pi Nv\overline{f}}{k(1-v^2)^{1/2}} \end{pmatrix}\psi$$

This is a slight modification of the Hamiltonian, so the eigenstates – the states that propagate without turning into each other – are only slightly different from the eigenstates in vacuum, that is, K_1^0 and K_2^0. A bit of algebra reveals that these states may be written

$$|K_1^{0\prime}\rangle = |K_1^0\rangle + r|K_2^0\rangle$$

$$|K_2^{0\prime}\rangle = |K_2^0\rangle - r|K_1^0\rangle$$

where the regeneration parameter, r (not to be confused with the mixing parameter r that is nearly unity for kaons), is a small number, typically of order 10^{-3},

$$r=\frac{-i\pi Nv(f-\overline{f})}{k(1-v^2)^{1/2}\gamma_1}[1/2-i(m_2-m_1)/\gamma_1]^{-1}$$

The expression has been simplified by noting that since the K_1^0 decays much faster than the K_2^0 , $\gamma_1 >> \gamma_2$.

If a neutral kaon beam travels a long distance, only K_2^0s are left. If the K_2^0s traverse a medium, their propagation must be analyzed in terms of the eigenstates in that medium. The K_2^0 is mostly $K_2^{0\prime}$, but with a small component of $K_1^{0\prime}$. These two pieces will acquire slightly different phases passing through the medium. When they exit, the states must be reanalyzed in terms of K_1^0 and K_2^0. This will reintroduce a component of K_1^0 of order r. The result is that an amplitude for K_1^0 will be generated proportional to

$$r\left[1-e^{(2i\delta m+\delta\gamma)L}\right]$$

where $L=l(1-v^2)^{1/2}/v$. We see then, that the amount of K_1^0 regenerated depends on the difference of the masses. It is thus possible to measure this difference which turns out to be extremely small compared to any other nonzero mass splitting.

An early measurement of the mass difference was made by F. Muller *et al.* at the Bevatron using regeneration techniques (**Ref. 7.4**). In addition to K_1^0s produced coherently in the forward direction, K_1^0s are produced away from the forward direction through the ordinary scattering process $K_2^0p\to K_1^0p$. This "diffractive" process produces particles mostly in the forward direction also, but not with such pronounced forward peaking as the coherent regeneration. Through the reaction

$\pi^- p \to K^0\Lambda$, Muller *et al.* generated a 670-MeV/c neutral kaon beam. A 30-inch propane bubble chamber was placed downstream where the surviving beam was purely K_2^0. The K_1^0 produced by the K_2^0 beam were detected by looking for charged pion pairs that reconstructed to the proper mass. By measuring the angular distribution of these K_1^0s it was possible to demonstrate the existence of the coherently regenerated beam. A first measurement of the mass difference was obtained:

$$(m_2 - m_1)/\gamma_1 = 0.85 \begin{array}{l} +0.3 \\ -0.25 \end{array}$$

The current values are $1/\gamma_1 = 0.8923 \pm 0.0022 \times 10^{-10}$ s and $m_2 - m_1 = 0.5349 \pm 0.0022 \times 10^{10}$ s^{-1}, giving $(m_2 - m_1)/\gamma_1 = 0.477$.

After the fall of parity invariance, it appeared that the combination of charge conjugation plus parity was still a good symmetry, as we assumed in the above analysis. There were (and are) solid theoretical reasons for believing that the combination of time reversal invariance, T, together with C and P gives a good symmetry, CPT. Thus if CP is a good symmetry, so is T.

If CP is a good symmetry, K_2^0 is strictly forbidden to decay into two pions. In 1964, Christenson, Cronin, Fitch, and Turlay observed the decay $K_2^0 \to \pi^+\pi^-$(**Ref. 7.5**). Another supposed symmetry had fallen. The experiment, carried out at the Alternating Gradient Synchrotron (AGS) at Brookhaven found that the CP-violating decay had a branching ratio of about 2×10^{-3}. Since most of the prominent decays of the longer-lived neutral kaon (which we henceforth refer to as K_L^0) have two charged particles in the final state, just as in the decay being sought, careful momentum measurements and particle identification were essential to separating $K_L^0 \to \pi^+\pi^-$ from the background.

The apparatus was a two-armed spectrometer, each arm of which had a magnet for momentum determination, scintillator for triggering on charged particles, a Čerenkov counter for discriminating against $e^\pm$ simulating $\pi^\pm$, and spark chambers for tracking the charged particles. A small but convincing signal was obtained for the CP violating decay. The experiment was soon repeated and confirmed at several laboratories.

If CP is broken, the physical eigenstates are linear combinations of K_1^0 and K_2^0. It turns out that as a result of CPT invariance only one (small) complex parameter is required to express the states:

$$|K_S^0\rangle = |K_1^0\rangle + \epsilon|K_2^0\rangle$$

$$|K_L^0\rangle = |K_2^0\rangle + \epsilon|K_1^0\rangle$$

where the normalizations of the states are good to order ϵ.

With CP violation, the equation for free propagation is

$$i\frac{\partial\psi}{\partial\tau} = \begin{pmatrix} \overline{m} - i\frac{\overline{\gamma}}{2} & \delta m - i\frac{\delta\gamma}{2} \\ \delta m^* - i\frac{\delta\gamma^*}{2} & \overline{m} - i\frac{\overline{\gamma}}{2} \end{pmatrix}\psi$$

where $\overline{m}$ and $\overline{\gamma}$ are still real, but δm and $\delta\gamma$ are complex. The off-diagonal δm corresponds to virtual K^0-$\overline{K}^0$ transitions while $\delta\gamma$ is due to real transitions. Thus $\delta\gamma$ is dominated by the $I = 0$ $\pi\pi$ state. With the Wu–Yang convention explained below, this amplitude is real and $\delta\gamma$ is thus nearly real as well. The δm term can be written $\delta m = \delta m_R + i\delta m_I$ with $\delta m_I << \delta m_R$. From the definitions of K_L^0 and K_S^0 we then find

$$\epsilon = \frac{i\delta m_I}{m_L - m_S + i\gamma_S/2}$$

This determines the phase of ϵ to be

$$\begin{aligned} arg(\epsilon) &= 90^\circ - \tan^{-1}\frac{\gamma_S}{2(m_L - m_S)} \\ &\approx 44^\circ \end{aligned}$$

Let us look at some of the details of the $K_L^0 \rightarrow \pi\pi$ decay. The 2π states can be decomposed into $I = 0$ and $I = 2$ components, since an $I = 1$ $\pi\pi$ state cannot have $J = 0$. If the final state pions did not interact with each other, CPT would provide a simple relation between the K^0 and $\overline{K}^0$ amplitudes:

$$\begin{aligned} \langle(2\pi)I = 0 \text{ stationary}|H_{\rm wk}|K^0\rangle &= A_0 \\ \langle(2\pi)I = 0 \text{ stationary}|H_{\rm wk}|\overline{K}^0\rangle &= -A_0^* \\ \langle(2\pi)I = 2 \text{ stationary}|H_{\rm wk}|K^0\rangle &= A_2 \\ \langle(2\pi)I = 2 \text{ stationary}|H_{\rm wk}|\overline{K}^0\rangle &= -A_2^* \end{aligned}$$

(Actually what we have written H_{wk} is really $i\,H_{wk}$ but this technicality will not affect the results.) The actual final states are not the "stationary" states above, but one in which the pions interact. Roughly speaking, because the pions are present only in the final state, they acquire half the usual strong interaction phase. Thus each amplitude is multiplied by $\exp(i\delta_I)$, where $I = 0$ or 2 is the isospin. When these results are assembled, the various $K^0 \rightarrow \pi\pi$ amplitudes turn out to be

$$\langle\pi^+\pi^-|H_{\rm wk}|K_L^0\rangle = \sqrt{2/3}e^{i\delta_2}(\epsilon{\rm Re}\,A_2 + i{\rm Im}\,A_2) + 2\sqrt{1/3}e^{i\delta_0}(\epsilon{\rm Re}\,A_0 + i{\rm Im}\,A_0)$$

$$\langle\pi^0\pi^0|H_{\rm wk}|K_L^0\rangle = 2\sqrt{1/3}e^{i\delta_2}(\epsilon{\rm Re}\,A_2 + i{\rm Im}\,A_2) - \sqrt{2/3}e^{i\delta_0}(\epsilon{\rm Re}\,A_0 + i{\rm Im}\,A_0)$$

$$\langle\pi^+\pi^-|H_{\rm wk}|K_S^0\rangle = \sqrt{2/3}(e^{i\delta_2}\mathrm{Re}\,A_2 + \sqrt{2}e^{i\delta_0}\mathrm{Re}\,A_0)$$

$$\langle\pi^0\pi^0|H_{\rm wk}|K_S^0\rangle = \sqrt{2/3}(\sqrt{2}e^{i\delta_2}\mathrm{Re}\,A_2 - e^{i\delta_0}\mathrm{Re}\,A_0)$$

These results can be simplified by observing the following. First, the much faster decay of the K_S^0 compared to $K^+ \to \pi^+\pi^0$ shows that $|A_0| >> |A_2|$. This is known as the $\Delta I = 1/2$ rule since the $\Delta I = 3/2$ interaction responsible for $K^+ \to \pi^+\pi^0$ is weaker than the $\Delta I = 1/2$ operator responsible for $K_S^0 \to \pi^+\pi^-$. Secondly, the phase of the K^0 state is still a matter of convention. It can be chosen so that A_0 is real. This is the convention of T. T. Wu and C. N. Yang. Thus we can drop the $\mathrm{Im}\,A_0$ terms, and terms of order $\epsilon A_2/A_0$. To compare the CP violating $K_L^0 \to 2\pi$ decay amplitudes to the CP-nonviolating $K_S^0 \to 2\pi$ amplitudes we define the ratios

$$\eta_{+-} = \frac{\langle\pi^+\pi^-|H_{\rm wk}|K_L^0\rangle}{\langle\pi^+\pi^-|H_{\rm wk}|K_S^0\rangle} = \epsilon + \epsilon'$$

$$\eta_{00} = \frac{\langle\pi^0\pi^0|H_{\rm wk}|K_L^0\rangle}{\langle\pi^0\pi^0|H_{\rm wk}|K_S^0\rangle} = \epsilon - 2\epsilon'$$

where

$$\epsilon' = \frac{1}{\sqrt{2}}\frac{\mathrm{Im}\,A_2}{A_0}\exp(i\pi/2 - i\delta_0 + i\delta_2)$$

With the Wu-Yang phase convention, ϵ measures the CP violation in the kaon states themselves, while ϵ' measures the CP violation in the decay. The measurement of the branching ratio B for $K_L^0 \to \pi^+\pi^-$, together with three more easily obtained numbers, gives the magnitude of η_{+-}:

$$|\eta_{+-}|^2 = \frac{\Gamma(K_L^0 \to \pi^+\pi^-)}{\Gamma(K_S^0 \to \pi^+\pi^-)} = \frac{B(K_L^0 \to \pi^+\pi^-)\Gamma(K_L^0 \to \mathrm{all})}{B(K_S^0 \to \pi^+\pi^-)\Gamma(K_S^0 \to \mathrm{all})} \approx (2.3 \times 10^{-3})^2$$

The analogous measurement for the decay into neutral pions is of course more difficult, but has been made.

To measure the phases of η_{+-} and η_{00} requires observing the interference between $K_L^0 \to \pi\pi$ and $K_S^0 \to \pi\pi$. This can be accomplished using a K_L^0 beam and regenerating a small amount of K_S^0, or by using a K^0 beam. In the latter case, one first sees the quickly decaying K_S^0 component. At the end, one sees only the CP violating K_L^0 decay (if care is taken to observe only the $\pi\pi$ final state!). In between, there is an interval when the contributions from K_S^0 and K_L^0 are comparable, and the interference can be measured. In Fig. 7.26 data obtained using the regenerator method are shown.

Figure 7.26: Data for $K_{L,S} \rightarrow \pi^+\pi^-$ as a function of the proper time after a K_L^0 beam has passed through a carbon regenerator. Curve A shows the detection efficiency as indicated on the right-hand scale. Curve B shows data for all values of the K momentum. The solid curve shows the shape expected in the absence of $K_L^0 - K_S^0$ interference. The interference is apparent and can be used to determine ϕ_{+-}. Curve C shows the data for a restricted interval of K momenta. The solid curve shows a fit including interference. [W. C. Carithers *et al.*, *Phys. Rev. Lett.* **34**, 1244 (1975)]

CP violation has been observed in only one other class of decays besides $K_L^0 \rightarrow \pi\pi$, namely $K_L^0 \rightarrow \mu\pi\nu$ and $K_L^0 \rightarrow e\pi\nu$. Aside from phase space considerations, these decays should be similar. From the $\Delta S = \Delta Q$ rule, one anticipates that the allowed decays to $\pi\mu\nu$ should be:

$$K^0 \rightarrow \pi^- \mu^+ \nu$$

$$\overline{K}^0 \rightarrow \pi^+ \mu^- \nu$$

It follows directly that

$$\delta = \frac{\Gamma(K_L^0 \rightarrow \pi^- \mu^+ \nu) - \Gamma(K_L^0 \rightarrow \pi^+ \mu^- \nu)}{\Gamma(K_L^0 \rightarrow \pi^+ \mu^- \nu) + \Gamma(K_L^0 \rightarrow \pi^- \mu^+ \nu)} \approx 2 Re\, \epsilon$$

Unlike the $K_L^0 \rightarrow \pi\pi$ decay, the decay process here is allowed even without CP violation. It is the small difference between two allowed rates that is due to CP violation. Thus very high statistics are required. The result can be compared to the measurement of the real part of ϵ obtained in the $K_L^0 \rightarrow \pi\pi$ decays. An early measurement of $K \rightarrow \pi e \nu$ was obtained by a group headed by Steinberger (Ref. 7.6). The analogous process, $K \rightarrow \pi\mu\nu$ was measured by a team led by M. Schwartz (Ref. 7.7). Data from a later experiment are shown in Fig. 7.25.

Because CP violation seems such a fundamental aspect of particle interactions, enormous efforts have been expended to measure the parameters η_{+-} and η_{00}. The values for the CP violation parameters cited in the 1988 *Review of Particle Properties* are

$$|\eta_{+-}| = (2.266 \pm 0.018) \times 10^{-3}$$

$$\phi_{+-} = \arg\, \eta_{+-} = 44.6^\circ \pm 1.2^\circ$$

$$|\eta_{00}| = (2.245 \pm 0.019) \times 10^{-3}$$

$$\phi_{00} = \arg\, \eta_{00} = 54^\circ \pm 5^\circ$$

$$\delta = (3.30 \pm 0.12) \times 10^{-3}$$

The results indicate that η_{+-} and η_{00} are very nearly equal, or, equivalently, ϵ' is nearly zero. This could be explained if all the CP violation were due to an interaction that changed strangeness by two units. All the CP violation then is in the K mass matrix and $\epsilon' = 0$. This is called the superweak model. The standard model of electroweak interactions, discussed in Chapter 12, makes predictions that are slightly different from the superweak model. It is thus possible to distinguish between the two with very precise measurements of the CP violating parameters. This can be accomplished by measuring $|\eta_{00}|^2$ and $|\eta_{+-}|^2$. Some uncertainties are reduced by taking the ratio

$$\frac{|\eta_{00}|^2}{|\eta_{+-}|^2} \approx 1 - 6\mathrm{Re}\frac{\epsilon'}{\epsilon} \approx 1 - 6\frac{\epsilon'}{\epsilon}$$

where ϵ'/ϵ is known to be nearly real because the phases of ϵ' and ϵ are quite similar. The most recent data from a major experiment at CERN (Ref. 7.8) indicate a small, nonzero value, $\epsilon'/\epsilon = 0.0033 \pm 0.0011$, and thus support the standard model rather than the superweak model.

EXERCISES

7.1 Derive the relation between the forward-scattering amplitude and the index of refraction by considering a plane wave of matter or light incident on a thin slab of material. Determine the shift in the phase of the wave passing through the material.

7.2 Show that the decay $\phi(1020) \to K_S^0 K_L^0$ is allowed but $\phi(1020) \to K_S^0 K_S^0$ and $\phi(1020) \to K_L^0 K_L^0$ are forbidden.

7.3 Verify the expression for the eigenstates of the neutral K system in matter. Estimate the size of the regeneration parameter in beryllium for a momentum of 1100 MeV, the conditions of the original CP violation experiment. Estimate f and $\overline{f}$ using the optical theorem and data for the K^+p and K^-p total cross sections.

7.4 A beam of K^0 is created at $t = 0$. Assuming CP conservation, what is the intensity of $\overline{K}^0$ in the beam as a function of the proper time? Plot the results for $|\Delta m|\tau_1 = 0, 1, 2, \infty$. See Camerini *et al.*, *Phys. Rev.* **128**, 362 (1962).

7.5 Consider a neutral kaon beam that is purely K^0 at $t = 0$. Show that the rate of decay into $\pi^+\pi^-$ as a function of the proper time, τ, is proportional to

$$e^{-\gamma_S\tau} + 2|\eta_{+-}|e^{-(\gamma_S+\gamma_L)/\tau/2}\cos[\phi_{+-} - (m_L - m_S)\tau] + e^{-\gamma_L\tau}|\eta_{+-}|^2.$$

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