HW #6 (129A), due Dec 6 , 4pm

1. Calculate the branching fraction of Z boson into $\nu\bar{\nu}$, e^+e^-, $\mu^+\mu^-$, $\tau^+\tau^-$, and all hadrons ($u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $c\bar{c}$, $b\bar{b}$) together with the correction factor due to the additional gluon emission, and compare them to the values in the PDG booklet.

2. Work out v from $G_F = 1/\sqrt{2}\ v^2$, $g = e/\sin\theta_W$, $g_Z = e/\sin\theta_W\cos\theta_W$, and show that the formulae $m_W = \frac{1}{2}gv$, $m_Z = \frac{1}{2}g_Z v$ do not quite work. On the other hand, using $\alpha(m_Z) = e^2(m_Z)/4\pi = 1/129$ instead of $1/137$, show that they work much better.

3. Show that the relation $m_W^2 = m_Z^2\cos^2\theta_W$ does not quite work. Instead, one has to take the effect of virtual top quark into consideration, and the correct relation is $m_W^2 = m_Z^2\rho\cos^2\theta_W$, with $\rho = 1 + 3G_F m_t^2/(8\sqrt{2}\,\pi^2)$. Here, the factor of three comes from the number of colors. Show that this relation works much better.

4. Plot the survival probability in the neutrino oscillation

$$P_{\rm surv} = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2}{4E} L, \tag{1}$$

as a function of the cosine zenith angle for atmospheric neutrinos. Assume that the neutrinos are produced at the altitude of 20 km. Take the parameters $\sin^2 2\theta = 1$, $\Delta m^2 = 3\times 10^{-3}\mathrm{eV}^2$, and $E = 1$ GeV.