

Fall 2002

Physics 129A

SOLUTIONS

to

H/w #2

$$\textcircled{1} (1) H = c \alpha^i p^i + mc^2 \beta$$

$$[H, p^j] = c \alpha^i [p^i, p^j] + 0 = 0$$

Therefore $\vec{p}$ is conserved, i.e. for any evolving state $|\psi(t)\rangle$, $\langle \vec{p} \rangle \equiv \langle \psi(t) | \vec{p} | \psi(t) \rangle$ is independent of time.

$$\begin{aligned} (2) [H, L^i] &= [c \alpha^l p^l + mc^2 \beta, \epsilon_{ijk} x^j p^k] \\ &= c \alpha^l \epsilon_{ijk} [p^l, x^j p^k] = \\ &= c \alpha^l \epsilon_{ijk} ([p^l, x^j] p^k + x^j [p^l, p^k]) \\ &= c \alpha^l \epsilon_{ijk} (-i\hbar \delta^{lj} p^k) = -i\hbar c \epsilon_{ijk} \alpha^j p^k \neq 0 \end{aligned}$$

Now, $\Sigma_i \equiv \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}$ is a 4×4 matrix,

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i^x \\ \sigma_i^x & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ are also } 4 \times 4$$

- 2 -

We need to find the commutators of these matrices. It's easy to check that if matrices A and B are separated into square blocks (in our case we have 4×4 matrices separated into 4 2×2 blocks) then when multiplying these matrices we can treat the blocks as if they were just numbers, i.e. if

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \quad \text{where all } a\text{'s}$$

and b 's are themselves two by two matrices, then

$$AB = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix},$$

just as if a 's and b 's were just #'s (just don't reverse the order of a 's and b 's in this formula, because while #'s commute, matrices don't).

Using that little theorem, we find:

$$\begin{aligned} [\Sigma_j, \Sigma_i] &= \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix} \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} - \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix} \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & \sigma_j \sigma_i \\ \sigma_j \sigma_i & 0 \end{pmatrix} - \begin{pmatrix} 0 & \sigma_i \sigma_j \\ \sigma_i \sigma_j & 0 \end{pmatrix} = \begin{pmatrix} 0 & [\sigma_j, \sigma_i] \\ [\sigma_j, \sigma_i] & 0 \end{pmatrix} = \end{aligned}$$

- 3 -

$$= \begin{pmatrix} 0 & -2i\epsilon_{ijk}\sigma^k \\ -2i\epsilon_{ijk}\sigma^k & 0 \end{pmatrix} = -2i\epsilon_{ijk}\alpha^k$$

$$[\beta, \Sigma_i] = \begin{pmatrix} \sigma_i & 0 \\ 0 & -\sigma_i \end{pmatrix} - \begin{pmatrix} \sigma_i & 0 \\ 0 & -\sigma_i \end{pmatrix} = 0$$

Hence,

$$[H, \frac{\hbar}{2}\Sigma_i] = \frac{\hbar}{2}c p^j [\alpha^j, \Sigma_i] = c p^j (-2i)\epsilon_{ijk}\alpha^k \frac{\hbar}{2}$$

and therefore

$$[H, L^i + \frac{\hbar}{2}\Sigma^i] = 0,$$

so $\vec{J} = \vec{L} + \frac{\hbar}{2}\vec{\Sigma}$ is conserved.

Notice that I was completely careless about ~~putting~~ writing the indices as subscripts or superscripts. The reason for this is that as long as we are performing calculations with vectors in the ordinary Euclidean space, where the metric $g^{ij} = \delta^{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, for any vector $\vec{V}$ $V^i = g^{ij} V_j = \delta^{ij} V_j = V_i$, so it doesn't matter where to put the index and you can put them at random if you wish. You don't have this luxury if $g^{ij} \neq \delta^{ij}$.

- 4 -

(3) $\vec{\Sigma}$ doesn't contain the $\vec{x}$ operator, so it commutes with $\vec{p}$. Then it's easy to see that, for instance,

J_z doesn't commute with p_x , because

$$\begin{aligned} [J_z, p_x] &= [L_z, p_x] = [x p_y - y p_x, p_x] \\ &= [x p_y, p_x] = [x, p_x] p_y + x [p_y, p_x] \\ &= i \hbar p_y \neq 0 \end{aligned}$$

$$(4) \quad \vec{p} \cdot \vec{J} = \vec{p} \cdot \vec{L} + \vec{p} \cdot \vec{\Sigma}$$

$\vec{p} \cdot \vec{\Sigma}$ still contains only numbers and components of $\vec{p}$, so

$$[\vec{p} \cdot \vec{\Sigma}, \vec{p}] = 0. \text{ Then}$$

$$[\vec{p} \cdot \vec{J}, p^i] = [p^k L_k, p^i]$$

$$= \epsilon_{k\ell m} [p^k x^\ell p^m, p^i] \underset{\substack{\uparrow \\ \text{since } p\text{'s commute}}}{=} \epsilon_{k\ell m} p^k [x^\ell, p^i] p^m$$

$$= \epsilon_{k\ell m} (i \hbar) p^k \delta^{\ell i} p^m$$

$$\text{But } \epsilon_{k\ell m} p^k p^m = \epsilon_{k\ell m} p^m p^k \underset{\substack{\uparrow \\ \text{just related the indices}}}{=} \epsilon_{m\ell k} p^k p^m = -\epsilon_{k\ell m} p^k p^m$$

$$\text{So } \epsilon_{k\ell m} p^k p^m = 0 \text{ and hence}$$

$$[\vec{p} \cdot \vec{J}, p^i] = 0$$

(Note: actually, one can show that $p^k L_k$ itself is zero.)

- 5 -

This shows that operators $\vec{p} \cdot \vec{J}$ and p^i have a common orthonormal eigentasis, i.e. there is a basis for the space of all states (Hilbert space) such that each basis state is an eigenstate of both momentum and $\vec{p} \cdot \vec{J}$. Therefore we can characterize these basis states by specifying 4 eigenvalues: p_x, p_y, p_z , and $\vec{p} \cdot \vec{J}$.

Now, in the notes on relativistic quantum mechanics we were looking in section 3.2 for eigenstates of momentum (i.e. for plane waves = states with a definite momentum). We found that they were of the form

$$u_{\pm}(p) e^{i(\vec{p} \cdot \vec{x} - Et)/\hbar}, \quad u_{\pm} = \begin{pmatrix} \sqrt{\frac{E+mc^2}{2mc^2}} \chi_{\pm}(\vec{p}) \\ \pm \sqrt{\frac{E-mc^2}{2mc^2}} \chi_{\pm}(\vec{p}) \end{pmatrix},$$

$$\chi_{+}(\vec{p}) = \begin{pmatrix} \cos \theta/2 \\ \sin \theta/2 e^{i\varphi} \end{pmatrix}, \quad \chi_{-}(\vec{p}) = \begin{pmatrix} -\sin \theta/2 e^{-i\varphi} \\ \cos \theta/2 \end{pmatrix},$$

$$\text{where } \vec{p} = |\vec{p}| (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$$

Now, it's important to distinguish numbers from operators and sometimes the notation isn't very clear, which means you have to guess from the context. In these formulas, for instance, $\vec{p}$ was just a vector, not the momentum operator. If we denote temporarily the momentum operator by $\hat{\vec{p}}$ to distinguish it from $\vec{p}$, the

- 6 -

precise statement of the above result is that for any vector $\vec{p}$ the states

$$u_{\pm}(p) e^{i(\vec{p} \cdot \vec{x} - Et)/\hbar}$$

are eigenstates of $\hat{\vec{p}}$ with eigenvalue $\vec{p}$ and of the Dirac Hamiltonian with eigenvalue $E \equiv \sqrt{|\vec{p}|^2 c^2 + m^2 c^4}$. Furthermore, it was claimed that these states are also eigenvectors of $\frac{\hbar}{2} \vec{p} \cdot \vec{\Sigma}$ and $\vec{p} \cdot \vec{J}$ with eigenvalues $\pm \frac{\hbar}{2} |\vec{p}|$. I will clarify and verify these statements and finally show that $\pm \frac{\hbar}{2} |\vec{p}|$ are the only eigenvalues of $\vec{p} \cdot \vec{J}$, as required in the problem.

1). I want to show that $\frac{\hbar}{2} \hat{\vec{p}} \cdot \vec{\Sigma} = \frac{\hbar}{2} \vec{\Sigma} \cdot \hat{\vec{p}} = \vec{J} \cdot \hat{\vec{p}} = \hat{\vec{p}} \cdot \vec{J}$

Proof. The first equality is obvious because $\hat{p}_i$ commutes with Σ_j .

To prove the others, I'll show that $\hat{p}^k L_k = L_k \hat{p}^k = 0$

Indeed, $\hat{p}^k L_k = \epsilon_{k\ell m} \hat{p}^k x^{\ell} \hat{p}^m$

But $\epsilon_{k\ell m} \neq 0$ only if k, ℓ , and m are all different. For example, we have a term $\epsilon_{312} \hat{p}^3 x^1 \hat{p}^2$. But it's cancelled by the term $\epsilon_{213} \hat{p}^2 x^1 \hat{p}^3$, because $\epsilon_{312} = -\epsilon_{213}$ and

$\hat{p}^3 x^1 \hat{p}^2 = \hat{p}^2 x^1 \hat{p}^3$ since $\hat{p}^2$, x^1 , and $\hat{p}^3$ all commute with each other. Other terms cancel in pairs in the same way. The same

story holds for $L_k \hat{p}^k$. So $\hat{\vec{p}} \cdot \vec{L} = \vec{L} \cdot \hat{\vec{p}} = 0$ and therefore

$$\vec{J} \cdot \hat{\vec{p}} = \frac{\hbar}{2} \vec{\Sigma} \cdot \hat{\vec{p}} = \frac{\hbar}{2} \hat{\vec{p}} \cdot \vec{\Sigma} = \hat{\vec{p}} \cdot \vec{J}.$$

- 7 -

2). Now I want to verify that $U_{\pm}(p)$ are indeed eigenvectors of $\vec{p} \cdot \vec{\Sigma}$. We found in the notes that the 2-component columns $\chi_{+}(\vec{p})$ and $\chi_{-}(\vec{p})$ were eigenvectors of $\vec{p} \cdot \vec{\sigma}$:

$$(\vec{p} \cdot \vec{\sigma}) \chi_{\pm}(\vec{p}) = \pm |\vec{p}| \chi_{\pm}(\vec{p})$$

Hence,

$$\begin{aligned}
 (\vec{p} \cdot \vec{\Sigma}) U_{\pm}(p) &= \vec{p} \cdot \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} U_{\pm}(p) = \begin{pmatrix} \vec{p} \cdot \vec{\sigma} & 0 \\ 0 & \vec{p} \cdot \vec{\sigma} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{E+mc^2}{2mc^2}} \chi_{\pm}(\vec{p}) \\ \pm \sqrt{\frac{E-mc^2}{2mc^2}} \chi_{\pm}(\vec{p}) \end{pmatrix} \\
 &\stackrel{\uparrow}{=} \begin{pmatrix} \vec{p} \cdot \vec{\sigma} \sqrt{} \chi_{\pm}(\vec{p}) \\ \vec{p} \cdot \vec{\sigma} (\pm \sqrt{}) \chi_{\pm}(\vec{p}) \end{pmatrix} = \begin{pmatrix} \pm |\vec{p}| \sqrt{} \chi_{\pm}(\vec{p}) \\ \pm |\vec{p}| (\pm \sqrt{}) \chi_{\pm}(\vec{p}) \end{pmatrix} \\
 &\text{using again the trick of treating} \\
 &\text{blocks as just numbers}
 \end{aligned}$$

$$= \pm |\vec{p}| \begin{pmatrix} \sqrt{} \chi_{\pm}(\vec{p}) \\ \pm \sqrt{} \chi_{\pm}(\vec{p}) \end{pmatrix} = \pm |\vec{p}| U_{\pm}(p)$$

This, of course, also proves that

$$\vec{p} \cdot \vec{J} U_{\pm} = \pm \frac{1}{2} \vec{p} \cdot \vec{\Sigma} U_{\pm} = \pm \frac{1}{2} |\vec{p}| U_{\pm}$$

3). Now I want to clarify the difference between

$\vec{p} \cdot \vec{J}$ and $\vec{p} \cdot \vec{J}$. First, note that if we fix $\vec{x}$ and t then $e^{i(\vec{p} \cdot \vec{x} - Et)/\hbar}$ is just a number, so

$U_{\pm}(p) e^{i(\vec{p} \cdot \vec{x} - Et)/\hbar}$ are also eigenstates of $\vec{p} \cdot \vec{J} = \pm \frac{1}{2} \vec{p} \cdot \vec{\Sigma}$

— 8 —

But $\vec{p} \cdot \vec{J} \neq \hat{\vec{p}} \cdot \vec{J}$, since $\vec{p}$ is a vector and $\hat{\vec{p}}$ is a vector operator, so are those states eigenstates of $\hat{\vec{p}} \cdot \vec{J}$ as well? It turns out yes, and here's why. We know that $\hat{\vec{p}} U_{\pm}(p) e^{i(\vec{p} \cdot \vec{x} - Et)/\hbar} = \vec{p} U_{\pm}(p) e^{i(\vec{p} \cdot \vec{x} - Et)/\hbar}$

(~~see beginning of page 6~~ see beginning of page 6; also, it's more or less obvious because $\hat{p}_i = -i\hbar \frac{\partial}{\partial x_i}$)

$$\begin{aligned} \text{So } \hat{\vec{p}} \cdot \vec{J} U_{\pm}(p) e^{i(\vec{p} \cdot \vec{x} - Et)/\hbar} &= \vec{J} \cdot \hat{\vec{p}} U_{\pm}(p) e^{i(\vec{p} \cdot \vec{x} - Et)/\hbar} \\ &= \vec{J} \cdot \vec{p} U_{\pm}(p) e^{i(\vec{p} \cdot \vec{x} - Et)/\hbar}, \text{ as we wanted to show.} \end{aligned}$$

In fact, this is actually the reason why the difference between $\hat{\vec{p}}$ and $\vec{p}$ is sometimes blurred — it doesn't matter what we use if we are acting on momentum eigenstates!

(But note that if we were acting on just $U_{\pm}(p)$, we would get different results: $\vec{J} \cdot \vec{p} U_{\pm}(p) = \pm \frac{\hbar}{2} |\vec{p}| U_{\pm}(p)$, but $\vec{J} \cdot \hat{\vec{p}} U_{\pm}(p) = 0$ since $\hat{\vec{p}} = -i\hbar \vec{\nabla}$ and $U_{\pm}(p)$ doesn't have any $\vec{x}$ -dependence.)

4). Finally, to show that $\pm \frac{\hbar}{2} |\vec{p}|$ are the only eigenvalues of $\vec{p} \cdot \vec{J}$ is the same as to show that $\pm |\vec{p}|$ are the only eigenvalues of $\vec{p} \cdot \vec{\Sigma}$. The easiest way to do this is to note that $\vec{p} \cdot \vec{\Sigma}$ is a 4×4 matrix and so has 4 eigenvalues,

— 9 —

which may be degenerate. So we'll be done if we show that both $|\vec{p}|$ and $-|\vec{p}|$ are doubly degenerate. But since

$(\vec{p} \cdot \vec{\sigma}) \chi_{\pm}(\vec{p}) = \pm |\vec{p}| \chi_{\pm}(\vec{p})$ it's easy to see that, for instance,

$\begin{pmatrix} \chi_+(\vec{p}) \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ \chi_+(\vec{p}) \end{pmatrix}$ are linearly indep. eigenstates

with eigenvalue $|\vec{p}|$ and ~~obviously~~ ^{similarly} for $-|\vec{p}|$. For example,

$$(\vec{p} \cdot \vec{\Sigma}) \begin{pmatrix} 0 \\ \chi_+ \end{pmatrix} = \begin{pmatrix} \vec{p} \cdot \vec{\sigma} & 0 \\ 0 & \vec{p} \cdot \vec{\sigma} \end{pmatrix} \begin{pmatrix} 0 \\ \chi_+ \end{pmatrix} = \begin{pmatrix} 0 \\ \vec{p} \cdot \vec{\sigma} \chi_+ \end{pmatrix} = |\vec{p}| \begin{pmatrix} 0 \\ \chi_+ \end{pmatrix}.$$

This proves that $|\vec{p}|$ and $-|\vec{p}|$ are both doubly degenerate, so we are done.

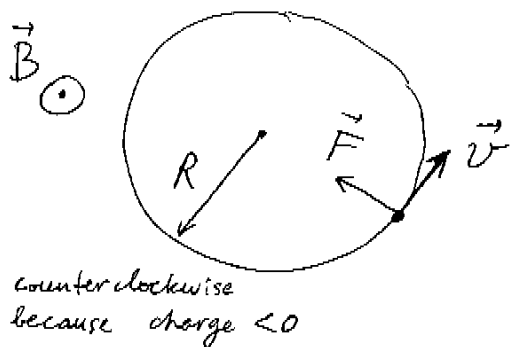
So the bottom line is: states $u_{\pm}(\vec{p}) e^{i(\vec{p} \cdot \vec{x} - Et)/\hbar}$ are simultaneous eigenstates of $\hat{\vec{p}}$, H , and $\vec{p} \cdot \vec{J}$ (and therefore, per discussion above, of $\hat{\vec{p}} \cdot \vec{J}$) and we could label them by specifying $\vec{p}$, E , and $\pm \frac{\hbar}{2} |\vec{p}|$ or even $\vec{p}$ and $\pm \frac{\hbar}{2}$ since E is completely determined by $\vec{p}$.

- ② The derivations of spin precession and Larmor frequencies can be found in numerous textbooks, so I'll reproduce them without much detail or explanation. The only thing worth mentioning is that since $\vec{p}$ is parallel to $\vec{v}$, the frequency of rotation $\vec{p}$ is equal to the frequency of the orbital motion of the electron.

— 10 —

Here's how we find the latter.

Picture,



Derivation,

$$F = \frac{+e}{c} \mathbf{v} \times \mathbf{B} \quad (\text{CGS})$$

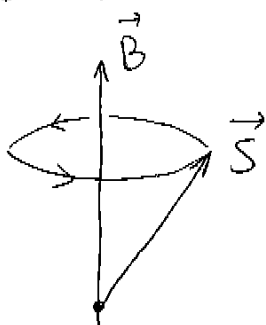
$$= ma = m \frac{v^2}{R}$$

$$\frac{+eB}{c} = \frac{m}{R} v = \frac{m}{R} \frac{2\pi R}{T} = m\omega$$

$$\boxed{\omega = \frac{eB}{mc}}$$

Here's how we find the frequency of the spin precession (classical treatment; QM gives the same result)

Picture.



$$\frac{d\vec{L}}{dt} = \vec{\tau}, \quad \vec{L} = \vec{S}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

The potential energy is $-\vec{\mu} \cdot \vec{B}$. Comparing with (39) in the notes, we see that

$$\vec{\mu} = g \frac{e\hbar}{2mc} \vec{S} = g \frac{e}{2mc} \vec{S} \quad \left(\vec{S} = \frac{\hbar}{2} \vec{\sigma} = \frac{\hbar}{2} \vec{S} \right)$$

Hence, $\frac{d\vec{S}}{dt} = g \frac{e}{2mc} \vec{S} \times \vec{B}$. Precession is described by

$\vec{S} = S_z \hat{z} + S_\perp \cos \omega t \hat{x} + S_\perp \sin \omega t \hat{y}$. Plug that in and use $\vec{B} = B \hat{z}$

$$S_\perp \omega (-\sin \omega t \hat{x} + \cos \omega t \hat{y}) = g \frac{e}{2mc} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ S_\perp \cos & S_\perp \sin & S_z \\ 0 & 0 & B \end{vmatrix} = \frac{ge}{2mc} S_\perp (B \sin \omega t \hat{x} - B \cos \omega t \hat{y})$$

$$\text{So } \boxed{\omega = - \frac{geB}{2mc}}$$

The minus sign indicates that the rotation is counterclockwise, as for $\vec{p}$. If $g=2$, frequencies match.

— 11 —

- ③. Let's first set up a general situation of one particle of mass m and momentum $\vec{p}_0 = p_0 \hat{x}$ colliding head-on with a particle of mass M originally at rest. Denote the final momenta $\vec{p}_1 = p_1 \hat{x}$ and $\vec{p}_2 = p_2 \hat{x}$ respectively (p_1 can be negative)

Setting $c=1$, we have:

$$\begin{cases} p_0 = p_1 + p_2 \\ M + \sqrt{p_0^2 + m^2} = \sqrt{p_1^2 + m^2} + \sqrt{p_2^2 + M^2} \end{cases}$$

Case 1. $m=0$, i.e. the first particle is a photon.

$$M + p_0 - |p_0 - p_2| = \sqrt{p_2^2 + M^2}$$

$p_1 = p_0 - p_2$ has to be negative, because otherwise the equation becomes $M + p_2 = \sqrt{M^2 + p_2^2}$, which has no solutions except $p_2 = 0$.

$$(M + 2p_0 - p_2)^2 = p_2^2 + M^2$$

$$(*) \quad 4p_0^2 + 4p_0(M - p_2) - 2Mp_2 = 0$$

$$p_0 = \frac{-(M - p_2) + \sqrt{(M - p_2)^2 + 2Mp_2}}{2} \quad (\text{minus sign would give } p_0 < 0)$$

$$\begin{aligned} p_0 &= \frac{1}{2} \left(\sqrt{p_2^2 + M^2} + p_2 - M \right) = \frac{1}{2} (E_2 + p_2 - M) \\ &= \frac{1}{2} M (\gamma + \gamma\beta - 1) \end{aligned}$$

We are told that the ~~velocity~~ velocity of ejected protons was $\frac{c}{10}$.

$$\text{So } \beta = \frac{1}{10}, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}} = 1.005, \quad p_0 = \frac{1}{2} 438 \text{ MeV} \left(1.005 + \frac{1.005}{10} - 1 \right)$$

- 12 -

The energy of the photon would then be

$$E_0 = \sqrt{p_0^2 + 0^2} = p_0 \approx \boxed{50 \text{ MeV}}$$

Now, to find the energy of a nitrogen nucleus when it's hit by a photon with this energy, rewrite (*) as

$$2p_0^2 + 2p_0 M = 2Mp_2 + 2p_0 p_2$$

$$p_2 = 2p_0 \frac{p_0 + M}{2p_0 + M} = 2p_0 \left(\frac{2p_0 + M}{2p_0 + M} - \frac{p_0}{2p_0 + M} \right)$$

$$= 2p_0 \left(1 - \frac{1}{2 + M/p_0} \right)$$

$$= 2 \times 50 \text{ MeV} \left(1 - \frac{1}{2 + \frac{14 \times 938 \text{ MeV}}{50 \text{ MeV}}} \right) = 100 \text{ MeV}$$

Since $p_2 \ll M = 14 \times 938 \text{ MeV}$, the kinetic energy is given by

the nonrelativistic expression $\frac{p_2^2}{2M} = \frac{(100 \text{ MeV})^2}{2 \times 14 \times 938 \text{ MeV}} = \boxed{378 \text{ keV}}$

Case 2. The first particle is a neutron. Since $m_n = m_p$, in the first scenario (ejection of protons) we have $M = m$ and the obvious solution is $p_0 = p_2$, $p_1 = 0$. So

$$p_0 = \gamma m \beta = 1.005 \times 938 \text{ MeV} \times \frac{1}{10} \approx 94.3 \text{ MeV}; E_0 = \gamma m = 943 \text{ MeV}$$

In the second scenario, the second particle is a nitrogen nucleus. $p_0 \ll m$, so we can assume the proton is non-relativistic. Even if it recoils with momentum $-p_0$, the resulting p_2 would still be much smaller than $M = 14m$, so the

- 13 -

nitrogen nucleus is also non-relativistic. In this approximation, our equations become:

$$p_0 = p_1 + p_2$$

$$M + m + \frac{p_0^2}{2m} = m + \frac{p_1^2}{2m} + M + \frac{p_2^2}{2M}$$

$$\frac{p_0^2}{m} = \frac{p_2^2 - 2p_0 p_2 + p_0^2}{m} + \frac{p_2^2}{14m}$$

$$15p_2^2 - 28p_0 p_2 = 0$$

$$p_2 = \frac{28p_0}{15} = \frac{28}{15} \times 94.3 \text{ MeV} = 176 \text{ MeV}$$

$$\text{Kinetic energy is } \frac{p_2^2}{2M} = \frac{(176 \text{ MeV})^2}{2 \times 14 \times 938 \text{ MeV}} = \boxed{1.2 \text{ MeV}}$$