Fall 2002

Physics 129A

SOLUTIONS to H/W #3

1. (a) $p \to e^+ \pi^0$ – forbidden (lepton and baryon number)

(b) $\mu^- \to e^- e^- e^+$ – forbidden (muon and electron lepton numbers; also, leptons don't participate in the strong interaction)

(c) $pp \to pn\pi^+$ – allowed

(d) $pn \to ppp\bar{n}$ – forbidden (electric charge)

(e) $\pi^+ p \to \Lambda K^+$ – forbidden (electric charge)

(f) $np \to \Sigma^0 K^+$ – forbidden (baryon number)

(g) $e^+ e^- \to pn$ – forbidden (baryon number; also, leptons don't participate in the strong interaction)

(h) $e^- p \to \pi^0 n$ – forbidden (lepton number)

(i) $\pi^- p \to K^0 n$ – forbidden (strangeness)

(j) $\pi^0 \to K^+ K^-$ – forbidden (energy conservation – to see this, view the reaction in the rest frame of π^0)

(2.) $r = \frac{\hbar c}{M} = 1.4 \times 10^{15}\,m$

---

(3) The book tells us that the energy loss of electrons is described by

$$E = E_1 e^{-x/X_0},$$

where E_1 – the initial energy, x – the path length, and X_0 – the radiation length. It also gives the radiation length for iron as $X_0^{iron} = 1.76$ cm and for lead as $X_0^{lead} = 0.56$ cm.

The article tells us that 1cm of platinum is equivalent to 1.96 cm of lead, i.e.

$$e^{-1cm/X_0^{Platinum}} = e^{-1.96cm/X_0^{lead}}$$

So $X_0^{Pl.} = X_0^{lead} \frac{1\,cm}{1.96\,cm} = 0.29\ cm$

Now, $\frac{-\Delta E}{d} = \frac{-(E_1 e^{-d/X_0^{Pl}} - E_1)}{d}$

$$= \left[\frac{1 - e^{-d/X_0^{Pl.}}}{d} \right] E_1 ,$$

so the expected slope is

$$\tan\theta \equiv \frac{-\Delta E/d}{E_1} = \frac{1 - e^{-1\,cm/0.29\,cm}}{1\,cm} = \boxed{0.97\ cm^{-1}}$$

(4.) States of ${}^{14}_{6}C$, ${}^{14}_{7}N$, and ${}^{14}_{8}O$ are states with $I_z = -1, 0$, and 1 respectively

States with $|I=1\ I_z=1\rangle$, $|1\ 0\rangle$, and $|1\ {-1}\rangle$ are related to one another by $SU(2)$ rotations in the isospin space, just like the three spin 1 states are related by $SU(2)$ rotations in ordinary space. Because the strong force is approximately symmetric with respect to these rotations, we expect that if, for instance, there is a state with $I = 1$, $I_z = 0$, and some values of other quantum numbers like spin or parity, then there are two more states with $I_z = \pm 1$ and the same values of the other quantum numbers, obtained from the original state by isospin rotations. Continuing the analogy with spin, if our Hamiltonian is symmetric with respect to the spin orientation (i.e. transformations changing the spin state of the system leave the Hamiltonian invariant), then if $\psi(\vec{r})\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$ is an eigenstate then $\psi(\vec{r})\,|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle$ is also an eigenstate. To see this, let $\mathcal{U}$ be the transformation that changes $|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$ to $|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle$ (it's actually just S_-). Now, given that $\hat{H}\,|\psi(\vec{r})\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle = E\,|\psi(\vec{r})\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$, we have

$$\hat{H}\,|\psi(\vec{r})\rangle|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle = \mathcal{U}\hat{H}\,\mathcal{U}^{-1}\,|\psi(\vec{r})\rangle|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle$$

$$= \mathcal{U}\hat{H}\,|\psi(\vec{r})\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle = E\,\mathcal{U}\,|\psi(\vec{r})\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$= E\,|\psi(\vec{r})\rangle|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle, \quad \text{qed}$$

where I used the invariance of the Hamiltonian:

$$\hat{H} = \mathcal{U}\hat{H}\,\mathcal{U}^{-1}, \quad \text{or} \quad \hat{H}\mathcal{U} - \mathcal{U}\hat{H} = 0$$

So the bottom line is: given that we have isospin symmetry, the $I=1$ states should always come in triplets, whose states have the same spin, parity etc.

Let's identify a few of these triplets. The lowest energy $|I=1\ I_z=0\rangle$ is the state of ${}^{14}_{7}N$ with energy 2312.798 keV and $J^{\text{parity}} = 0^+$. The other members of the triplet should be the lowest ~~[illegible]~~ energy $I=1$ states of ${}^{14}_{8}O$ and ${}^{14}_{6}C$. If we are right, these states should also have $J^P = 0^+$. Looking at the table we see that that's indeed the case, so our first triplet is:

$I_z = -1$	$I_z = 0$	$I_z = 1$
${}^{14}_{6}C$	${}^{14}_{7}N$	${}^{14}_{8}O$
0 keV, 0^+	2312 keV, 0^+	0 keV, 0^+

The fact that the energies are different shouldn't bother us because the energies are given with respect to the ground states of the ~~[illegible]~~ corresponding isotopes. But we do expect the energy <u>differences</u> between members of different triplets to be <u>approximately</u> the same for the three isotopes.

— 6 —

The next $|I=1\ I_z=0\rangle$ state is ${}^{14}_{7}N$, 8062 keV, 1^-

Following the same logic, we identify another triplet:

$I_z = -1$	0	+1
C	N	O
6093 keV, 1^-	8062 keV, 1^-	5173 keV, 1^-

(the value for I is for some reason not given for ${}^{14}_{6}C$, but there are no 1^- states of C that have even remotely similar energy)

The next triplet is, by the same token:

6589 keV, 0^+	8618 keV, 0^+	5920 keV, 0^+

and we can continue that way for the rest of the table.

Now, to identify singlet states is even easier. Of course, there are no $I=0$ states of C and O, because they have $I_z = \pm 1$. Looking at the table for N, we see the following $I=0$ levels:

0 keV 1^+, 3948 keV 1^+, 4915 keV 0^-, 5105 keV 2^-, 5691 keV 1^-,

etc.