Physics 129A Fall 2002

SOLUTIONS to the Midterm.

(1.) (1) Color was introduced in order to explain why the spin/flavor wavefunction for baryons should be symmetric rather than antisymmetric. If there was no color, the spin/flavor wavefunction would be completely antisymmetric by the Fermi–Dirac statistics. Consider, however, Δ^{++} (which consists of three up quarks) with spin $\frac{3}{2}$ along the z-direction. Its flavor wavefunction is, of course, just $|u\rangle_1 |u\rangle_2 |u\rangle_3$, which is totally symmetric. But the spin part must be $|\uparrow\rangle_1 |\uparrow\rangle_2 |\uparrow\rangle_3$ to give $S_z = \frac{3}{2}$, so it's also symmetric. Contradiction! To resolve it, the color quantum number was proposed. We postulate that each quark can carry one of three colors: red, green, or blue.

Colors can transform into one another by performing $SU(3)$ rotations in the color space. We further postulate that any bound state must be color-neutral, i.e. a singlet under $SU(3)$. But it turns out there's only one possible color wavefunction that is a singlet, and it happens to be completely antisymmetric. Thus the Fermi-Dirac statistics requires the spin/flavor wavefunction to be completely symmetric – exactly what we need, as we saw in the above example.

(2) The most convincing evidence for the existence of color comes from experiments comparing the cross-sections for

$$e^+e^- \to \mu^+\mu^-$$

and

$$e^+e^- \to \text{quark antiquark}$$

In the high energy approximation, where we can ignore the effect of the masses, the cross-section is determined only by the charge of the outgoing particles (the charge determines the coupling strength). Thus we can easily calculate the ratio

$$\frac{\sum_q \sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

because we know the charge of each quark. If we assume there's no color the ratio comes out 3 times less than what's measured experimentally. If color exists the ratio increases by a factor of 3 because for each flavor of quark, three separate final states are now possible instead of one, corresponding to three possible colors of the final quarks. Thus, assuming color exists, the theoretical prediction matches with experiment.

Note: these are very detailed explanations. On the midterm, you didn't have to provide that much detail, but you were still required to capture the essense of the arguments.

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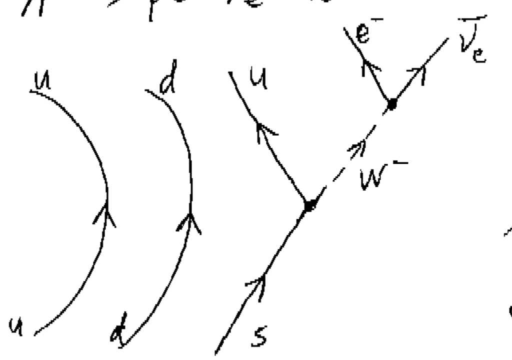
(2) $\Lambda^0 \to p e^- \bar{\nu}_e$ is a weak strangeness-changing process.



This means we have an extra factor of $\sin^2\theta_w$ in Γ.

Using dimensional analysis and inserting a typical 3-body phase-space factor, we estimate:

$$\Gamma = \frac{1}{192\pi^3} G_F^2 \sin^2\theta_w \, Q^5 ,$$

where $Q = M_{\Lambda^0} - M_p$ is the size of the phase-space.

We have: $G_F = 1.17 \times 10^{-5}\,\mathrm{GeV}^{-2}$

$\theta_W = 13.1^\circ$

$M_{\Lambda^0} = 1115.6\,\mathrm{MeV}$

$M_p = 938.79\,\mathrm{MeV}$

Inserting these into Γ, we get:

$$\Gamma = 2.0 \times 10^{-10}\,\mathrm{eV}$$

Using $\hbar = 6.5822 \times 10^{-16}\,\mathrm{eV\,s}$, we get the decay rate:

$$\boxed{\frac{\Gamma}{\hbar} = 3.0 \times 10^{5}\,\frac{1}{s}}$$

and the lifetime:

$$\tau = \frac{\hbar}{\Gamma} = 3.3 \times 10^{-6}\,s$$

To compare this with the data, we find the total lifetime of Λ^0:

$$\tau_{tot} = 2.63 \times 10^{-10}\,s$$

and the branching ratio $\frac{\Gamma_i}{\Gamma_{tot}} \equiv \frac{\tau_{tot}}{\tau_i} = 8.32 \times 10^{-4}$,

so the experimental value for the lifetime is

$$\tau_i = \frac{2.63 \times 10^{-10}\,s}{8.32 \times 10^{-4}} = 3.2 \times 10^{-7}\,s$$

and the decay rate

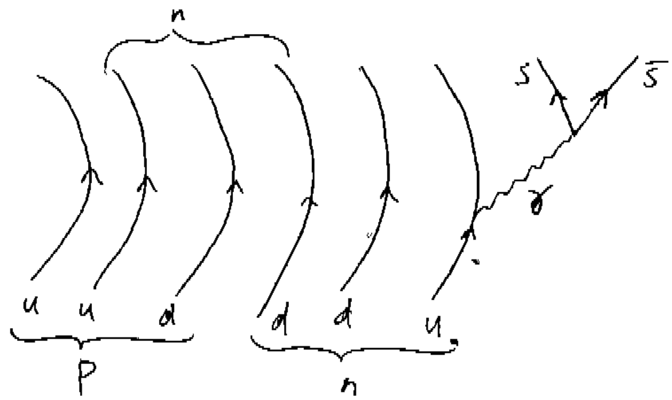
$$\boxed{\frac{1}{\tau_i} = 3.2 \times 10^{6}\,\frac{1}{s}}$$

It differs from our estimate by an order of magnitude.

(3) (a) $\mu^- \to e^- \nu_e \bar{\nu}_e$ forbidden by muon number conservation

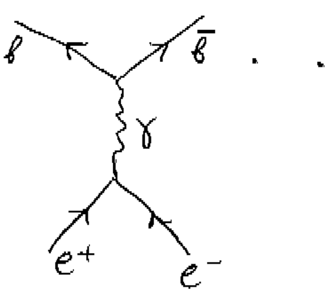
(b) $\cancel{A} pn \to \Lambda^0 K^+ n$ allowed

$p = uud$, $n = udd$, $\Lambda^0 = uds$, $K^+ = u\bar{s}$



Other diagrams are possible, of course.

(c) $e^+ e^- \to b \bar{b}$ allowed

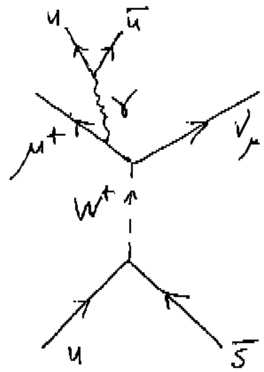


(d) $\omega^0 \to \pi^0 \pi^0$ forbidden by Pauli exclusion principle

ω^0 has $J=1$, π^0 has $J=0$, so the final state would need to have $L=1$, which is impossible for two bosons

(e) $K^+ \to \pi^0 \mu^+ \nu_\mu$ allowed

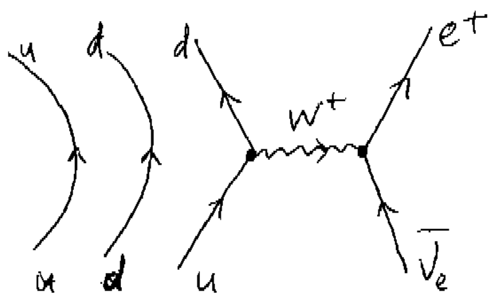
$K^+ = u\bar{s}$, $\pi^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$



(f) $\Sigma^0 \to K^- p$ forbidden by energy conservation

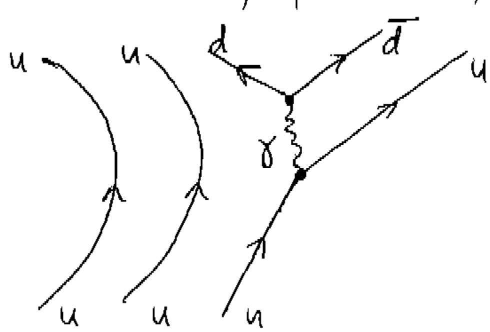
(g) $p \to \mu^+ \pi^0$ forbidden by muon number conservation

(h) $\bar{\nu}_e p \to e^+ n$ allowed



(i) $\Delta^{++} \to p \pi^+$ allowed

$\Delta^{++} = uuu$, $p = uud$, $\pi^+ = u\bar{d}$



(j) $e^+ e^- \to \mu^+ \nu_\mu$ forbidden by charge conservation