Midterm Exam (221B), due Mar 23, 4pm

1. Use Thomas–Fermi model of atoms to answer the following questions.

   (a) Solve the differential equation numerically and obtain a plot of Thomas–Fermi function $\chi(x)$ for $x < 20$.

   (b) Based on this result, argue why higher l orbitals have higher energies with a given principal quantum number n.

   (c) Calculate the total binding energy of an atom.

   (d) Show that the "radius" of atoms depends only weakly on Z, consistent with the empirical fact. The radius is naively infinite in this model because the charge density extends smoothly to infinity. Define the radius R instead to contain $Z - 1$ electrons

   $$Z - 1 = \int_0^R 4\pi r^2 dr \rho(r). \tag{1}$$

   Plot R in Å as a function of Z.

2. Consider nuclei as a sphere with a uniform charge density.

   (a) Calculate the Rutherford scattering cross section of an electron off a nucleus together with the form factor.

   (b) Comparing it to the data in J. B. Bellicard et al, *Phys. Rev. Lett.*, **19**, 527 (1967), estimate the size of Calcium nuclei.

   (c) Comparing it to the data in J. B. Bellicard et al, *Phys. Rev. Lett.*, **19**, 242 (1967), estimate the size of lead nuclei.

   (d) Discuss A dependence of the size of nuclei.

3. Consider the Helium atom with two electrons. We would like to understand how important the correlation between two electrons is. Use the trial (spatial) wave function

   $$\psi(\vec{x}_1, \vec{x}_2) = N e^{-Z' r_1/a_0} e^{-Z' r_2/a_0} (1 + c_1 u + c_2 t^2) \tag{2}$$

   to calculate the total binding energy using the variational method. Compare the results with (a) Z' only, (b) Z' and c_1, and (c) Z', c_1, c_2. The spin part of the wave function is totally anti-symmetric ($S = 0$ combination), and N is the overall normalization constant. Here, $u = r_{12}/a_0$, $t = (r_1 - r_2)/a_0$, and $a_0 = \hbar^2/me^2$ is the Bohr radius. The experimental value for the Helium binding energy is 78.605 eV.