221B HW #3, due Feb 4 (Fri), 4pm

1. Consider a nucleus as a sphere with a uniform charge density.

   (a) Calculate the Rutherford scattering cross section of an electron by a nucleus together with the form factor using the first Born approximation.

   (b) Comparing it to the data in J. B. Bellicard et al, *Phys. Rev. Lett.*, **19**, 527 (1967), estimate the size of Calcium nuclei, using the location of dips in the cross sections.

   (c) Comparing it to the data in J. B. Bellicard et al, *Phys. Rev. Lett.*, **19**, 242 (1967), estimate the size of lead nuclei, using the location of dips in the cross sections.

   (d) Discuss A dependence of the size of nuclei.

   Note Even though the electrons used in these experiments are relativistic, it is still true that the form factor is the Fourier transform of the charge density to the extent that we ignore the nuclear recoil. Be careful, however, to use the electron momentum and energy with the relativistic formula $E = cp$, and regard the form factor a function of momentum trasnfer $q^2 = 2p^2(1 - \cos\theta)$.

2. Consider the scattering problem by the Yukawa potential

$$V = V_0 \frac{e^{-r/a}}{r} \tag{1}$$

   in three dimensions.

   (a) Calculate the scattering amplitude and the total cross section using Born approximation.

   (b) Discuss the validity of Born approximation for the Yukawa potential, by requiring

$$\frac{2m}{\hbar^2} \left| \int d\vec{x} \frac{e^{ikr}}{4\pi r} V(\vec{x}) e^{ikz} \right| \ll 1. \tag{2}$$

   (c) Show that the total cross section is smaller than the "geometric cross section" $\sim 4\pi a^2$ when Born approximation is valid independent of the momenta.