Take-Home Midterm Exam (221B), due March 18, 4pm

1. Consider two electrons in the np orbital. Answer the following question.

   (a) Show that the Coulomb repulsion between two electrons has the form

   $$\langle np^2|\frac{e^2}{r_{12}}|np^2\rangle = \begin{cases} F^0 + \frac{2}{5}F^2 & {}^1\mathrm{S} \\ F^0 + \frac{1}{25}F^2 & {}^1\mathrm{D} \\ F^0 - \frac{1}{5}F^2 & {}^3\mathrm{P} \end{cases} \tag{1}$$

   Note that all information about the radial wave function is contained in the integrals F^0 and F^2.

   (b) Test the relationship against the low-lying energy levels of group IV (electrons) and group VI (holes) elements. (For states split by the spin-orbit interactions, use their weighted averages.) Explain why the test fails for very low and very high Z.

2. For $Z = 13$, use the Thomas–Fermi model to work out the energy levels of $3p$, $3d$, $4s$, $4p$, $4d$, $4f$, $5s$, $5p$, $6s$ orbitals numerically. Compare the ordering to the excited spectrum of aluminum.

3. Thomson's plum pudding model of the atoms was excluded by the Rutherford's experiment. Assume a Gaussian distribution

   $$\rho_N(\vec{x}) = \frac{Z}{(2\pi\sigma)^{3/2}}e^{-\vec{x}^2/2\sigma^2} \tag{2}$$

   of the positive charge in the atom with $\sigma \approx a_B$ instead of a point-like nucleus $\rho_N(\vec{x}) = Z\delta(\vec{x})$. Calculate the scattering cross section of the alpha particle off the atom (neglect the electrons that are so light that they get thrown out by bulldozing alpha particle) using the Born approximation, and compare to that with the point-like nucleus.