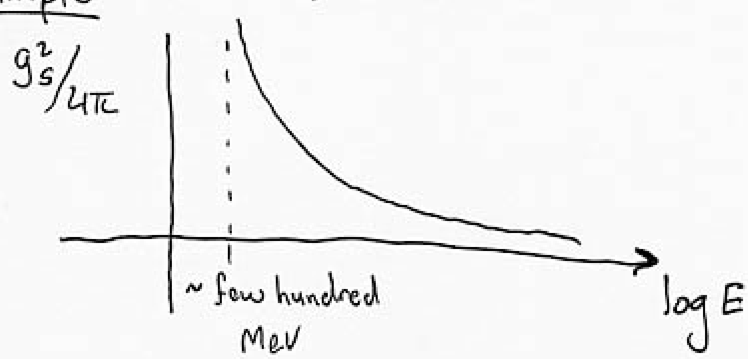
① . Py 229 C 6/12/2005

Anomaly continued:

Lagrangian has a symmetry (which leads to a conservation law) but quantisation violates the symmetry and hence the conservation law is lost.

Example QCD



Classical Lagrangian $\rightarrow \int d^4x \, \frac{1}{g^2}\left(-\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}\right)$

- no intrinsic energy scale
- no dimensionful parameter

$\Rightarrow$ scale invariance, conformal symmetry

But after quantisation

no scale invariance, $\exists$ intrinsic energy scale

In our 1+1 D E&M example

particle number is conserved classically but not conserved quantum mechanically

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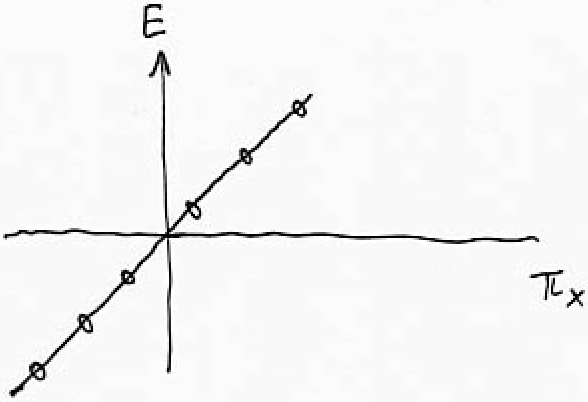
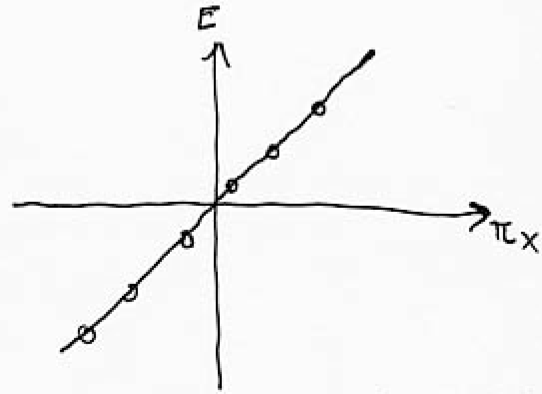
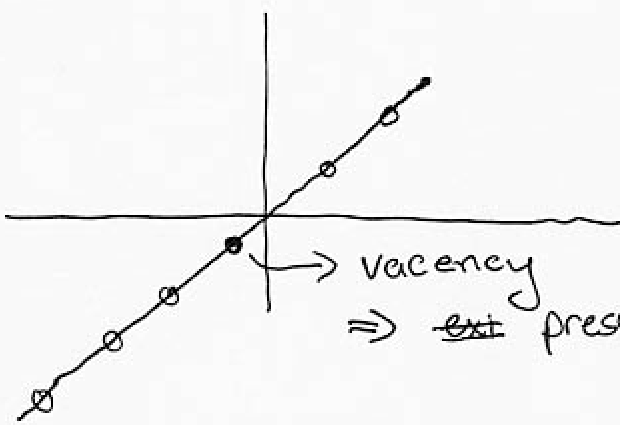


$A_x = 0$



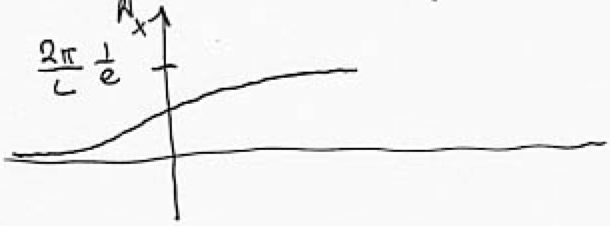
$A_x \neq 0$ levels shifted

If $A_x = \frac{2\pi}{L} \frac{1}{e}$ then:



$\Rightarrow$ ~~ext~~ presence of an anti-particle

Where does the energy come from?



$$E_x = -\dot{A}_x - \cancel{\partial_x \phi}$$

$$\text{Energy} = \frac{1}{2}\int E^2 \, dx$$

Thermal bath can provide energy to create anti-particle
$\Rightarrow$ violates the conservation of particle number

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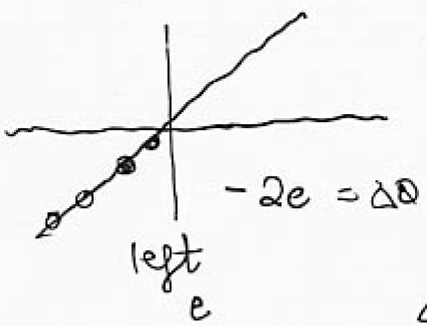
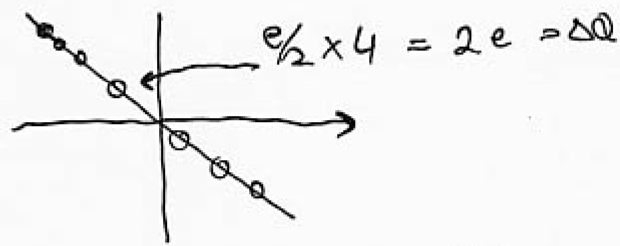
- change conservation of charge?

In 1+1 D E+M w/ only one left-handed fermion of charge e $\Rightarrow$ theory is inconsistent

If we include 4 right handed fermions of charge $e/2$

Then if A_x $0 \to \frac{2\pi}{L}\frac{1}{e} \times 2$





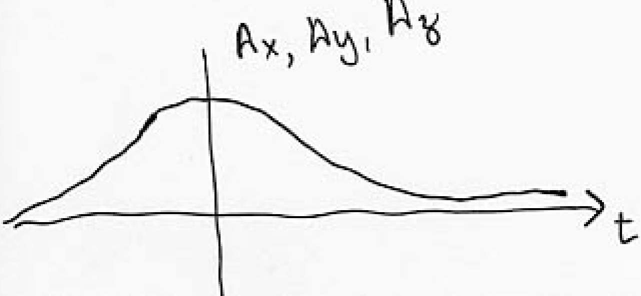
$\Delta Q = 0$ but $\Delta N_{particles} \neq 0$

How does this work in Weak Interactions?

3+1 D Weak Interaction:

$\begin{pmatrix} u \\ d \end{pmatrix}^G_L \quad \begin{pmatrix} u \\ d \end{pmatrix}^R_L \quad \begin{pmatrix} u \\ d \end{pmatrix}^B_L \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$

$\Rightarrow$ also leads to the non-conservation of particle number.



$\to$ all the energy levels shift by one unit $\to$ particle creation

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$$\partial_\mu j_2^\mu = \overset{\#\text{generation}}{3} \frac{g^2}{64\pi^2} W^a_{\mu\nu} W^a_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma} \neq 0$$

$$\partial_\mu j_B^\mu = 3 \frac{g^2}{64\pi^2} W^a_{\mu\nu} W^a_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}$$

$$\text{Lepton }\# = \#e^- + \#\mu^- + \#\tau^- + \#\nu_e + \#\nu_\mu + \#\nu_\tau$$
$$- \#e^+ - \#\mu^+ + \#\tau^+ + \#\bar{\nu}_e + \#\bar{\nu}_\mu + \#\bar{\nu}_\tau$$

Comment

1+1D E+M $\pi_1(U(1)) = \mathbb{Z}$

3+1D SU(2) Weak $\pi_3(SU(2)) = \mathbb{Z}$

$\Big\}$ same homotopy group so same mathematical structure

Process in ~~eq~~ early universe

$$q_1 q_1 q_1 l_1 \quad q_2 q_2 q_2 l_2 \quad q_3 q_3 q_3 l_3 \longleftrightarrow 0$$

$$q_1 q_1 q_1 q_2 q_2 q_3 l_3 \longleftrightarrow \bar{l}_1 \bar{l}_2 \bar{q}_3 \bar{q}_3 \bar{q}_3$$

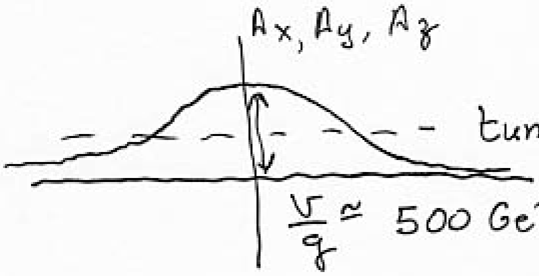
The rate of the anomaly induced particle non-conservation

$$\Gamma \sim 20\, \alpha_W^5 T \quad . \quad \alpha_W = \frac{g^2}{4\pi} \left(\frac{e^2}{4\pi \sin^2\theta_W} \right) \sim \frac{1}{129}$$

$$H \sim \frac{T^2}{M_{Pl}}$$

$\Rightarrow$ In equilibrium $T_e \leq T \leq 10^{12}$ GeV

Below T_c of BEC of Higgs Boson



$t_{ritium} \longrightarrow e^+ \bar{\nu}_\mu \bar{\nu}_\tau$ is possible but exponentially suppressed by WKB factor $\propto e^{-8\pi^2/g^2}$ (Instanton)

n n p

$\tau > 10^{150}$ years.

Chemical equilibrium in early Universe

Q_L	μ_Q
u_R	μ_u
d_R	μ_d
L_L	μ_L
e_R	μ_e
H	μ_H

For bosons

$$n = g \frac{1}{\pi^2} T^3 \, Li_3 (e^{\beta\mu})$$

For Ferimons

$$n = g \frac{1}{\pi^2} T^3 (-Li_3(-e^{-\beta\mu}))$$

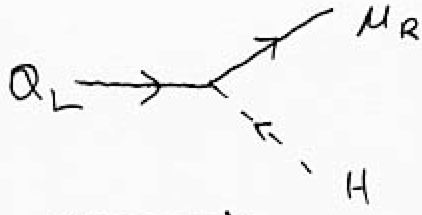
where $Li_3(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^3}$

When $\mu \ll T$

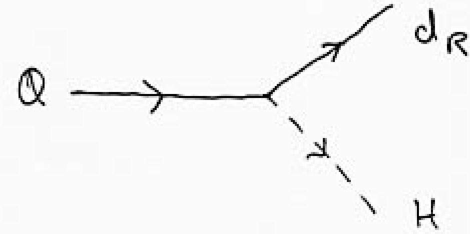
$$n_{bosons} = g \frac{\zeta(3)}{\pi^2} T^3 + g \frac{1}{\pi^2} T^3 \frac{\pi^2}{6} \overbrace{(e^{\beta\mu} - 1)}^{\beta\mu} + O(e^{\beta\mu}-1)^2$$

$$= \frac{g\,\zeta(3)}{\pi^2} T^3 + \frac{g}{6} T^2 \mu + O(\mu^2)$$

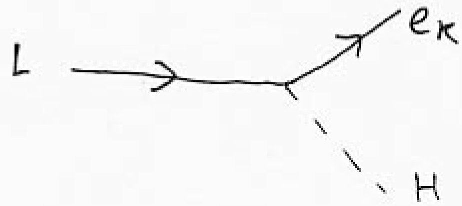
$$n_{fermions} = g \frac{3}{4} \frac{\zeta(3)}{\pi^2} T^3 + \frac{g}{12} T^2 \mu + O(\mu^2)$$



$\mu_Q = \mu_u - \mu_H$

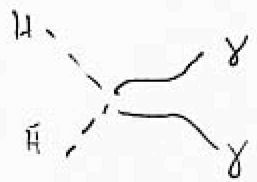


$\mu_Q = \mu_d + \mu_H$



$\Big\{ \mu_L = \mu_e + \mu_H$

also require total charge of universe = 0
hypercharge $Y = Q - I_3$



$\mu_H + \mu_{\bar{H}} = 0$

$$\frac{1}{2}(n_H - n_{\bar{H}}) + \frac{1}{6}(n_Q - n_{\bar{Q}}) + \left(\frac{-1}{2}\right)(n_L - n_{\bar{L}}) + \frac{2}{3}(n_u - n_{\bar{u}}) + \frac{-1}{3}(n_d - n_{\bar{d}}) = 0.$$

4 eqn's $\underset{\substack{\text{hypercharge}\\ \text{density}}}{Y} = \frac{1}{12}T^2\left(8\mu_H + 6\mu_Q + 12\mu_u - 6\mu_d - 6\mu_L - 6\mu_e\right) = 0$

5th eqn. Anomaly $QQQL \leftrightarrow 0$ for each generation

$3\mu_Q + \mu_L = 0$

One unknown, B−L is conserved, undetermined so far

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$$B_{\text{density}} = \frac{1}{3}\cdot\frac{1}{12} T^2 \left(36\mu_Q + 18\mu_u + 18\mu_d\right)$$

$$L_{\text{density}} = \frac{1}{12} T^2 \left(12\mu_L + 6\mu_e\right).$$

B-L density fixed

N generations $= 3$ in SM

M # Higgs $= 1$ in SM

– Plugged into Mathematica to solve

$$\mu_u = \frac{M-2N}{4N}\mu_H \qquad \mu_Q = -\frac{M+2N}{4N}\mu_H$$

$$\mu_d = -\frac{M+6N}{4N}\mu_H \qquad \mu_L = \frac{3M+6N}{4N}\mu_H$$

$$\mu_e = \frac{3M+2N}{4N}\mu_H$$

Thus:

$$B_{\text{density}} = \frac{4(M+2N)}{13M+22N}(B-L) = 0.354\,(B-L) \quad (\text{SM})$$

$$L_{\text{density}} = -\frac{9M+14N}{13M+22N}(B-L) = -0.646\,(B-L) \quad (\text{SM})$$

Initial B+L ~~does~~ not help unless $B-L \neq 0$

Create L initially
no B

$\Big\}$ leptogenesis $\longrightarrow$ baryogenesis

$\Uparrow$ anomaly in SM

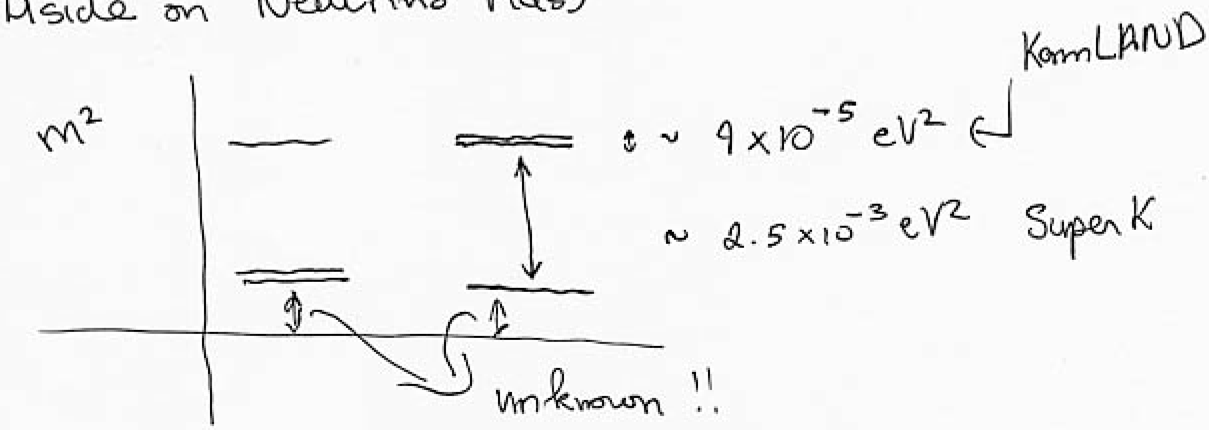
Leptogenesis

Create $L < 0$ initially   how ?

$\Downarrow$ anomaly

$B > 0$

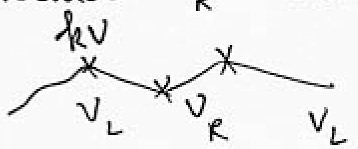
Aside on Neutrino Mass



Dirac Neutrino ?   or   Majorana Neutrino ?

$\hookrightarrow$ Repeat what is done for e, ...

Introduce ν_R (new species)



$m_\nu = h_\nu v$, with $h_\nu \rightsquigarrow 10^{-13}$

$\hat{}$

$m_e = h_e v$

Majorana ~~Neu~~ Mass:

$\frac{v^2}{\Lambda} \sim 0.1\ eV$

$\Lambda \sim 10^{14} - 10^{15}\ GeV.$

No Light new particle d.o.f.s

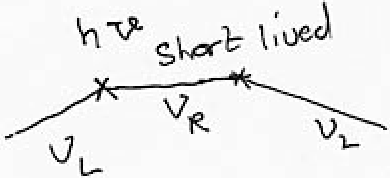
Seesaw Mechanism

$\Lambda \uparrow \quad m_\nu \downarrow$

Implementations in realistic models.

do introduce ν_R, doesn't feel any of the SM forces but heavy, allowed to have a mass independent of Higgs BEC so $M_{\nu_R} \not\propto v$ and easily $M_R \gg v$

$\Delta E \Delta t \sim 1$



$$m_\nu = \frac{(h_\nu v)^2}{M_{\nu_R}}$$

$$\begin{pmatrix} 0 & h_\nu v \\ h_\nu v & M \end{pmatrix} \longrightarrow M, \quad \frac{(h_\nu v)^2}{M}$$