HW #1 (229C), due Sep 23, 4pm

1. Using the Friedmann equation

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3} G_N \rho - \frac{k}{R^2}, \tag{1}$$

discuss how old the Universe is. The most accurate determination of the Hubble constant is $H_0 = 71\text{km/s/Mpc} \times (1.00^{+0.04}_{-0.03})$ from the WMAP data on cosmic microwave background anisotropy (see, *e.g.*, `http://map.gsfc.nasa.gov/m_uni/uni_101expand.html`).

   (a) Assume the matter-dominated universe with flat geometry, and work out the age of the universe, and compare it with the estimated age of the old stars in the globular clusters in our Milky Way galaxy (see, *e.g.*, `http://www.eso.org/outreach/press-rel/pr-2004/pr-20-04.html`).

   (b) Discuss if the combination of matter and curvature can resolve the paradox.

   (c) Show that the paradox can be resolved if you allow for the vacuum energy without introducing the curvature term.

2. (optional) If you know general relativity, use the Friedmann–Robertson–Walker metric

$$ds^2 = c^2 dt^2 - R(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right) \tag{2}$$

to work out the Ricci tensors

$$R^\phi_\phi = R^\theta_\theta = R^r_r = -\left(2\frac{k}{R^2} + 2\frac{\dot{R}^2}{c^2 R^2} + \frac{\ddot{R}}{c^2 R} \right) \tag{3}$$

$$R^t_t = -3\frac{\ddot{R}}{c^2 R} \tag{4}$$

and the resulting Friedmann equation. The convention for the signature is $(+, -, -, -)$.