HW #5 (229C), due Dec 2, 4pm

1. Solve the Boltzmann equation for a cold relic

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{eq}^2). \tag{1}$$

First rewrite the equation for the variable $x = m/T$ and the yield $Y = n/s$. Solve it numerically for the S-wave $\langle \sigma v \rangle = \sigma_0$ (constant independent of x) or the P-wave $\langle \sigma v \rangle = \sigma_0 x^{-1}$. You may use the "typical" values $g_* = 100$, $\sigma_0 = 10^{-10}$ GeV^{-2}, $m = 1000$ GeV. Plot the x-dependence of the yield and compare it to the equilibrium yield to see how it deviates from the equilibrium. Compare the asymptotic value of the yield to the analytic estimate

$$Y(\infty) = \frac{H(m)}{s(m)} \frac{(n+1)x_f^{n+1}}{\sigma_0} \tag{2}$$

and determine the approximate value of the freeze out temperature x_f.

2. For the obtained yield, work out the contribution of the relic to the current energy density Ω.

Optional If you know the quantum field theory and Feynman diagrams, calculate the annihilation cross section of binos via the interaction Lagrangian

$$\mathcal{L}_{int} = \sqrt{2} g' \tilde{e}_R^* \tilde{B} e_R + h.c. \tag{3}$$

Pay a careful attention to the fact that the bino is a Majorana fermion and show that the process is dominantly P-wave.