What's new with Goldstone?

Haruki Watanabe, Hitoshi Murayama
+ Tomáš Brauner

CERN theory colloquium
December 11, 2013

arXiv:1203.0609, 1302.4800, 1303.1527
all published in *PRL*

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Nobel Prizes and Laureates

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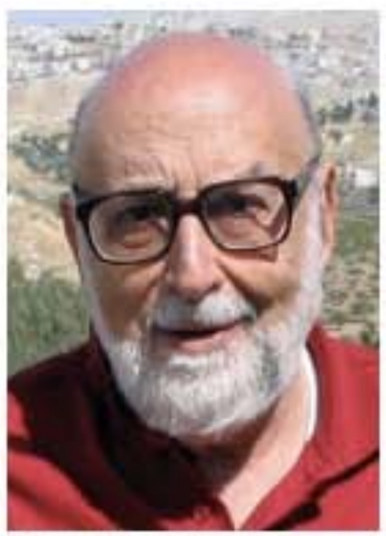
▶ François Englert
▶ Peter W. Higgs

All Nobel Prizes in Physics
All Nobel Prizes in 2013

The Nobel Prize in Physics 2013
François Englert, Peter W. Higgs

The Nobel Prize in Physics 2013



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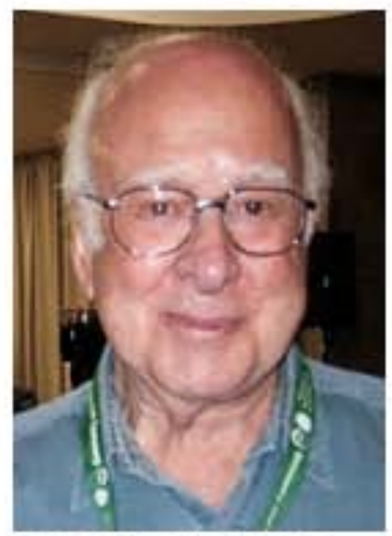

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Peter W. Higgs

The Nobel Prize in Physics 2013 was awarded jointly to François Engle Peter W. Higgs *"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of predicted fundamental particle, by the ATLAS and CMS experiments a CERN's Large Hadron Collider"*

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Scanned at the American Institute of Physics

50-year old puzzle

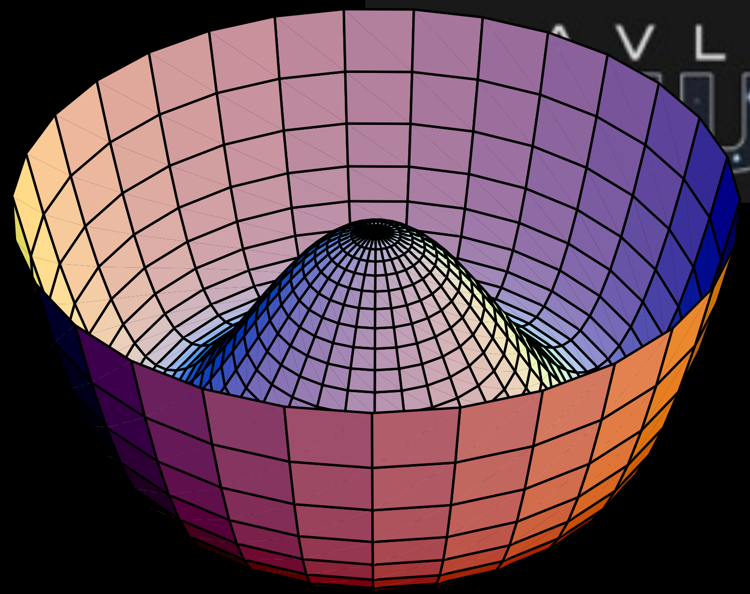
- Goldstone's theorem
  - for every spontaneously broken symmetry, there is a massless excitation
  - $E = c\,p$ (linear dispersion relation)
- in Heisenberg ferromagnet
  - 1 gapless excitation for 2 broken symm
  - $E \propto p^2$ (quadratic dispersion relation)
- *What is going on?*

Outline

- Introduction (lengthy)
- Main points = simple points
- Formalism –Internal Symmetries–
- Previously known results
- Redundancies (brief)
- Massive NGBs (brief)
- Conclusion

Introduction

SSB is Ubiquitous



- χ-symmetry in strong interactions (QCD)
- crystals (spatial translations)
- ^{4}He superfluid, BEC (particle number)
- ^{3}He superfluid, spinor BEC, neutron stars, kaon condensation, color superconductivity, etc (a rich variety of symmetries)
- superconductors, Higgs (gauge invariance)
- *what is the underlying unified description?*

Goldstone's theorem

- When a continuous symmetry *G* is spontaneously broken to its subgroup *H*, there are massless bosons $E=c\,p$ for every broken generators.

$$\langle \pi^a(p) | j^b_\mu(0) | 0 \rangle = f_\pi \delta^{ab} p^\mu$$

- $n_{\rm NGB}=n_{\rm BG}$
- assumes Lorentz invariance and positive definite metric of the Hilbert space

longitudinal

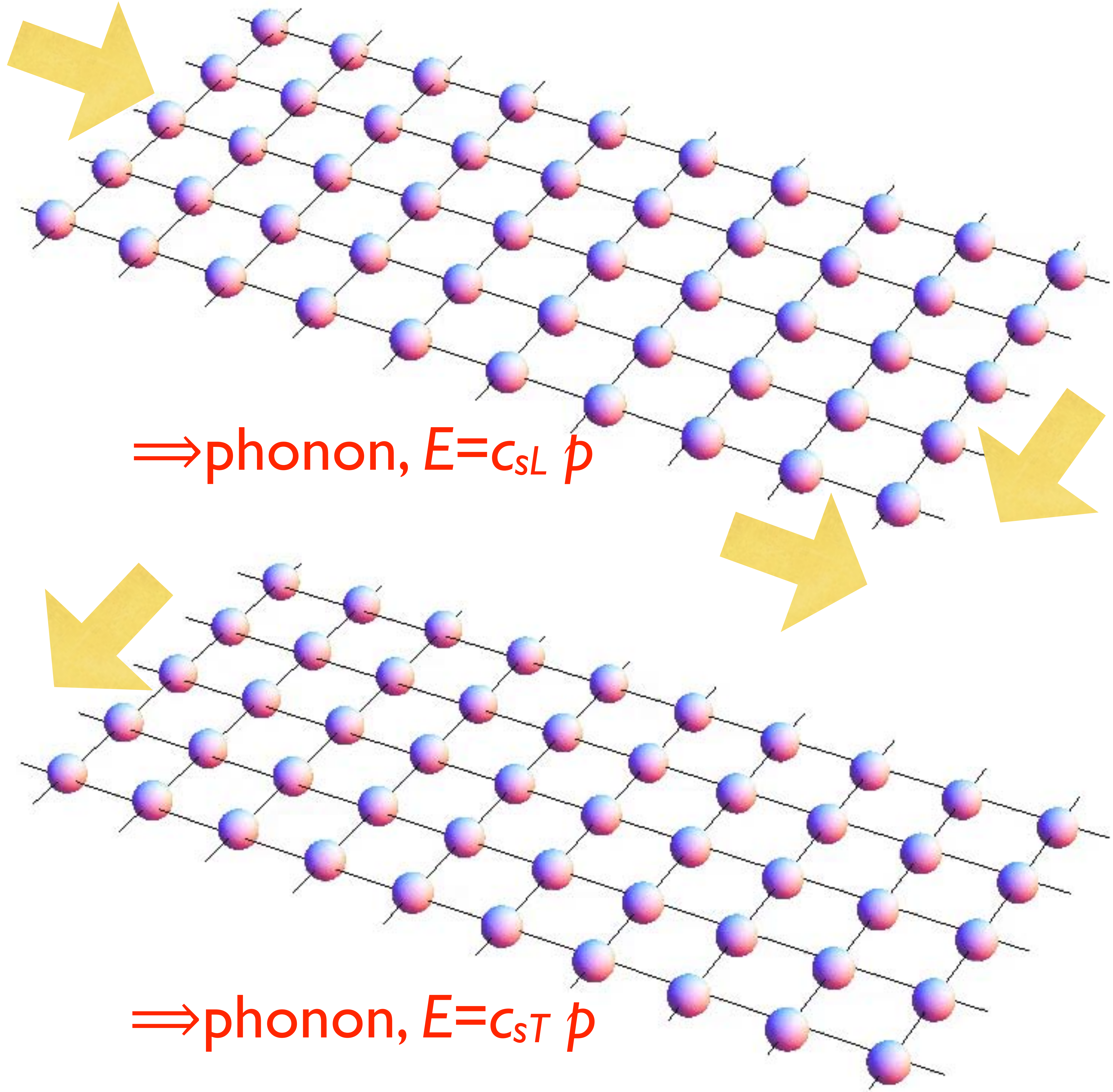
crystal

$G=\mathbb{R}^2$

$H=\mathbb{Z}^2$

$G/H=T^2$

transverse

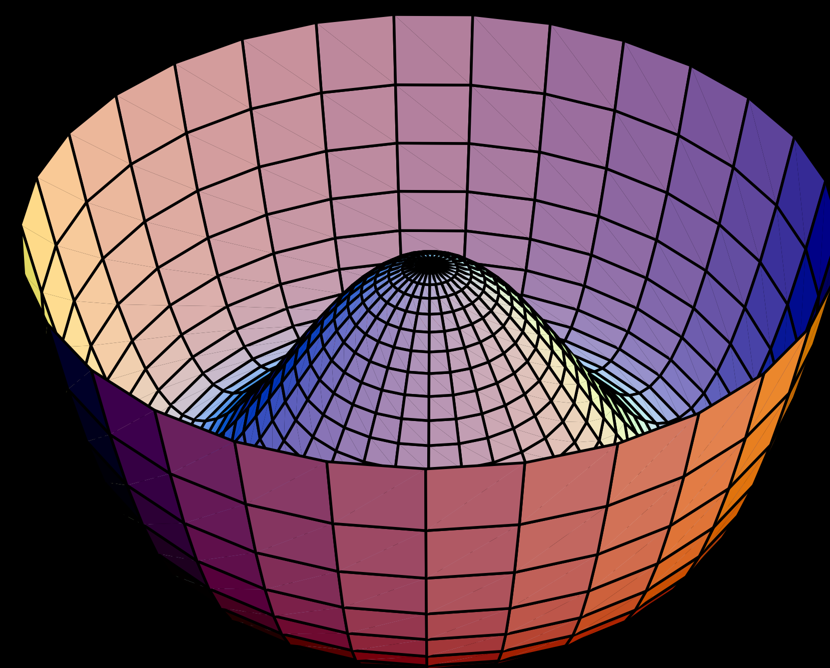


Particle numbers

- U(1) symmetry

$$\psi(x) \to e^{i\theta}\psi(x)$$

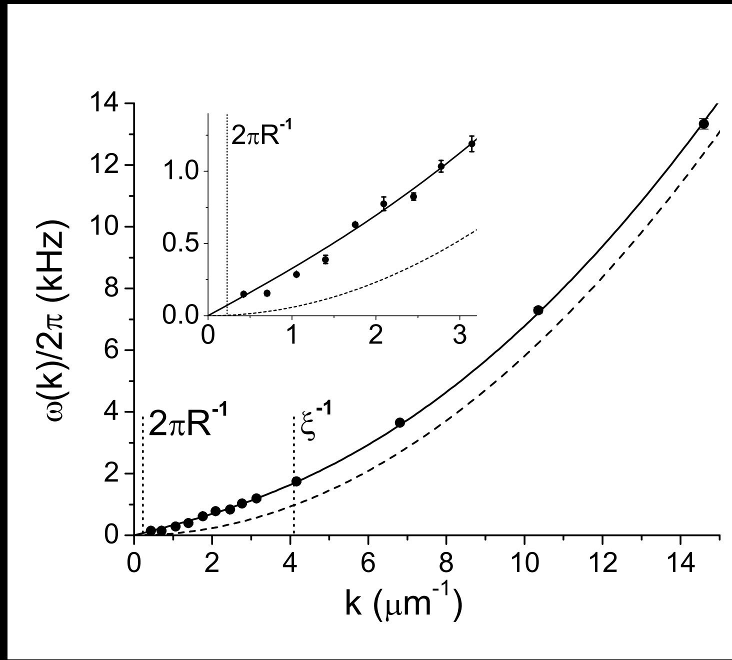
$$N = \int dx \psi^* \psi$$

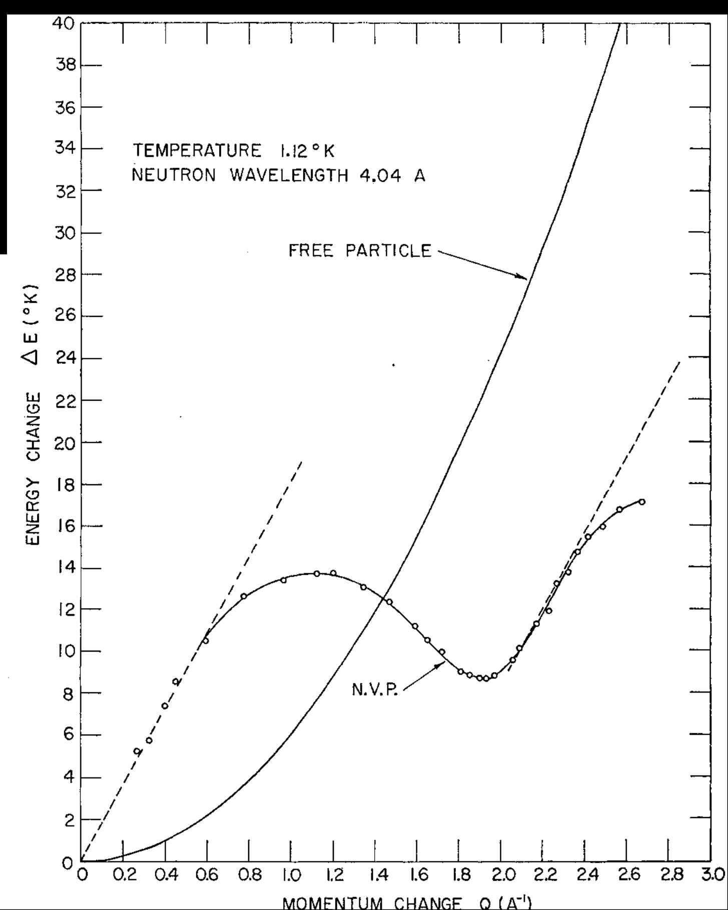


- Ginzburg-Landau theory

$$V = -\mu\psi^*\psi + \lambda(\psi^*\psi)^2 \qquad \langle 0|\psi|0\rangle \neq 0$$

- G=U(1), H=0
- ^{4}He superfluid
- scalar BEC





Heisenberg models

$$H = +J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j$$

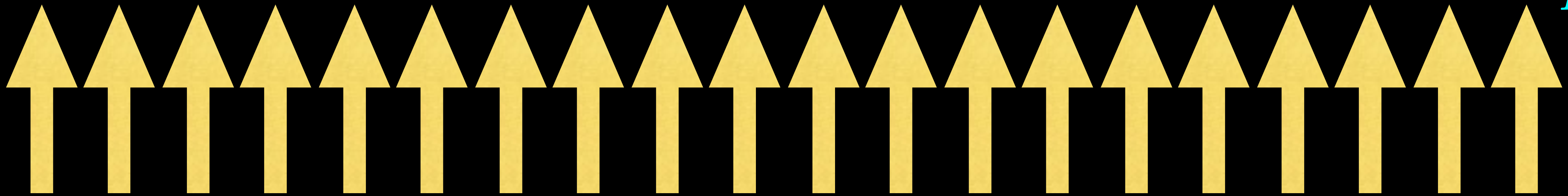
2 NGBs

$E \propto p$

$$H = -J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j$$

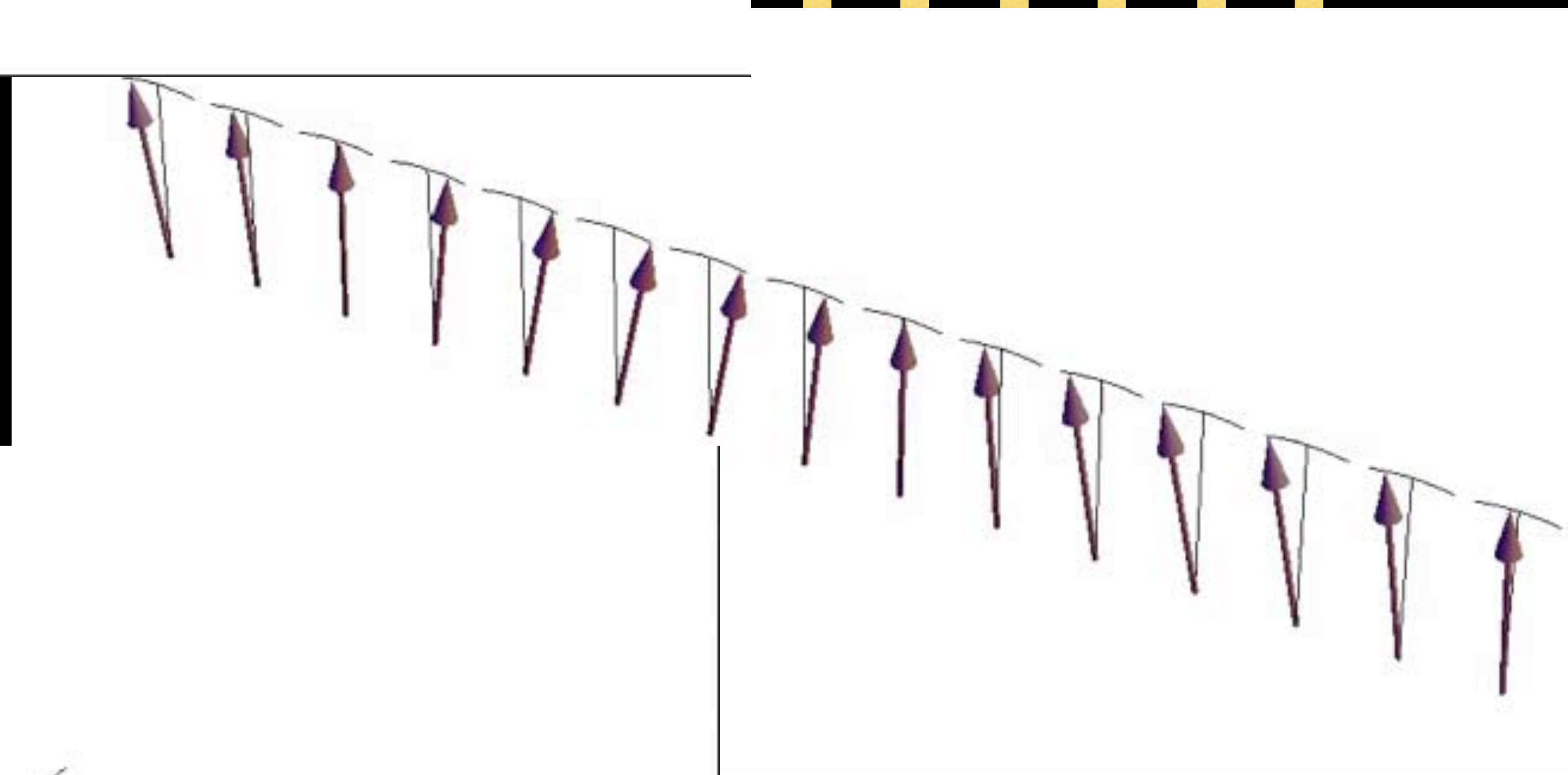
1 NGB

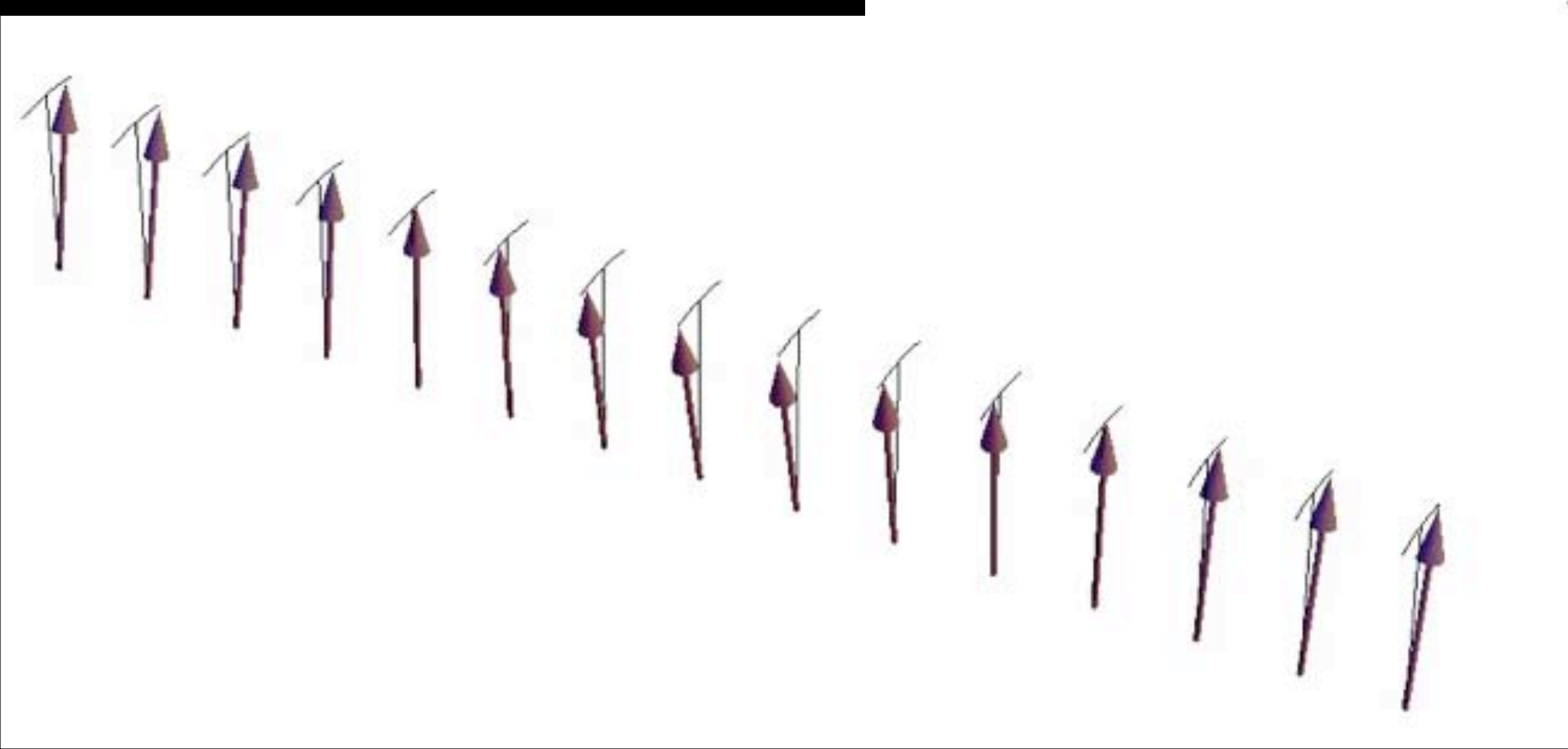
$E \propto p^2$



Both G/H = SO(3)/SO(2) = S^2

s?





No!

experiment

- Dispersion relations can be tested experimentally
- specific heat
  - $E \propto p \Rightarrow C_V \propto T^3$
  - $E \propto p^2 \Rightarrow C_V \propto T^{3/2}$
- Plot $C_V/T^{3/2}$ vs $T^{3/2}$

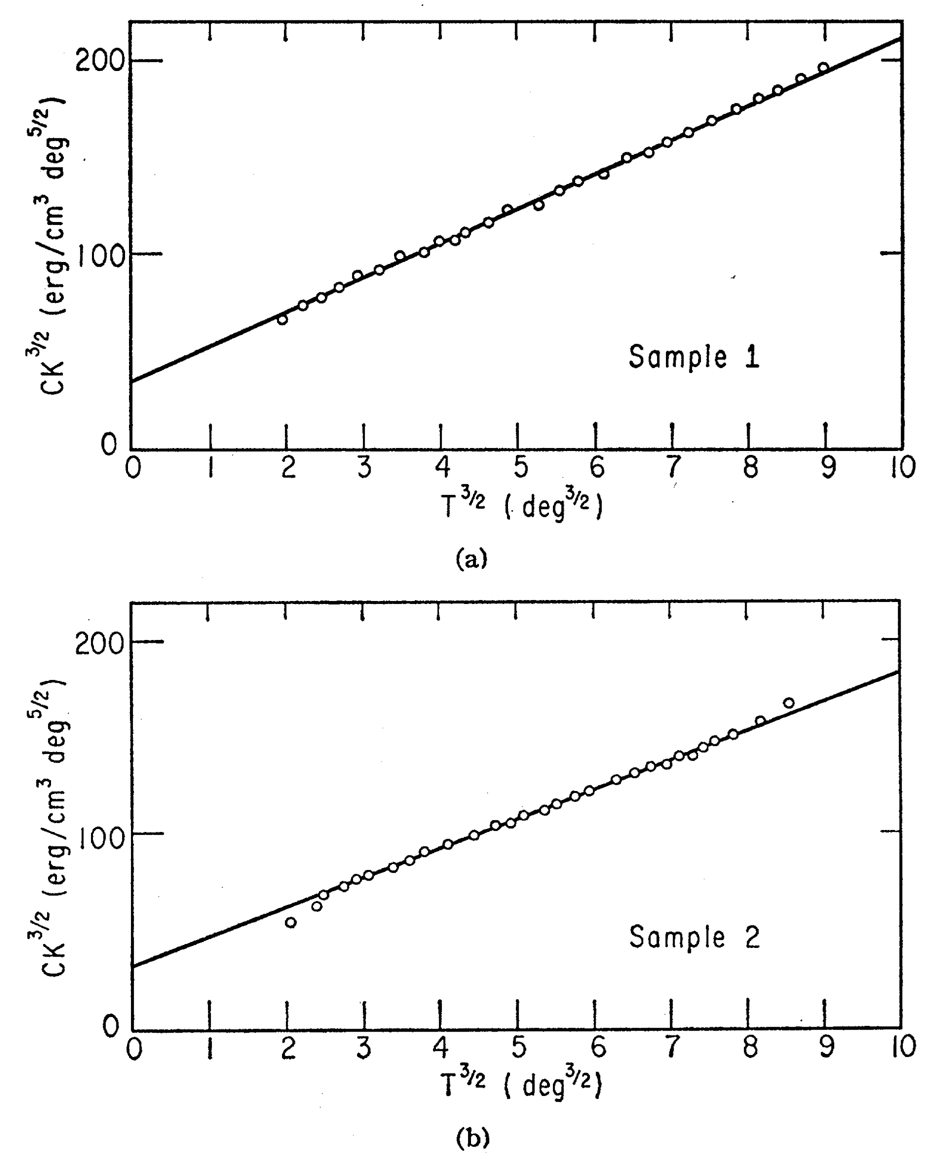
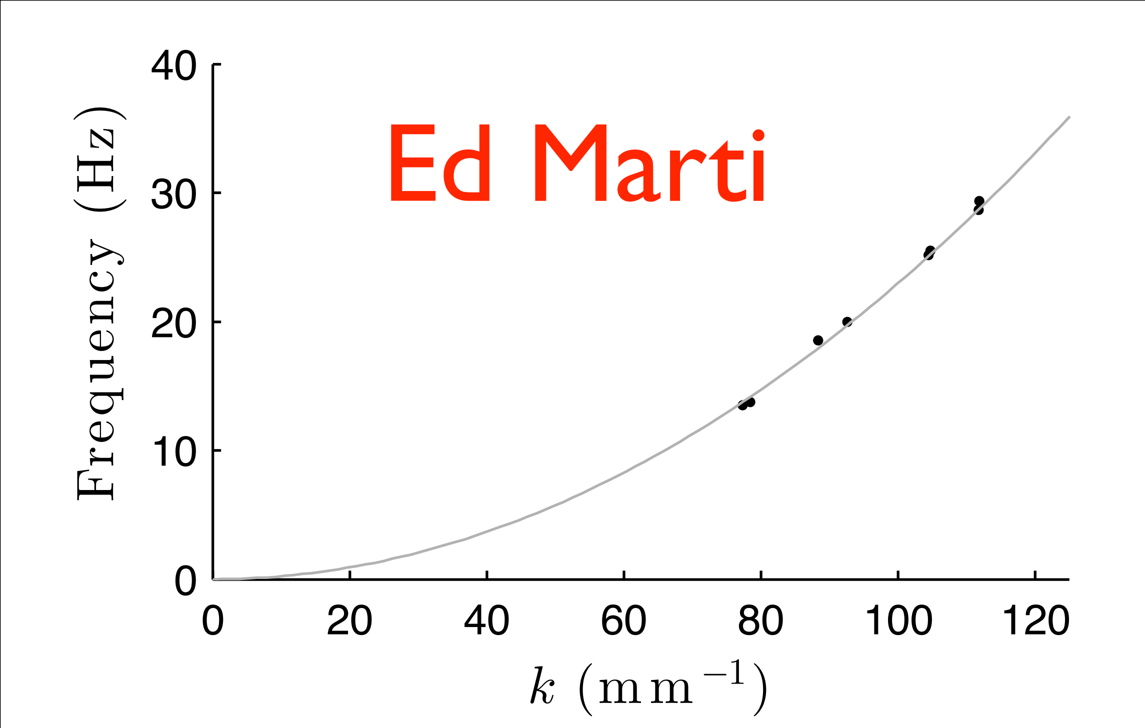
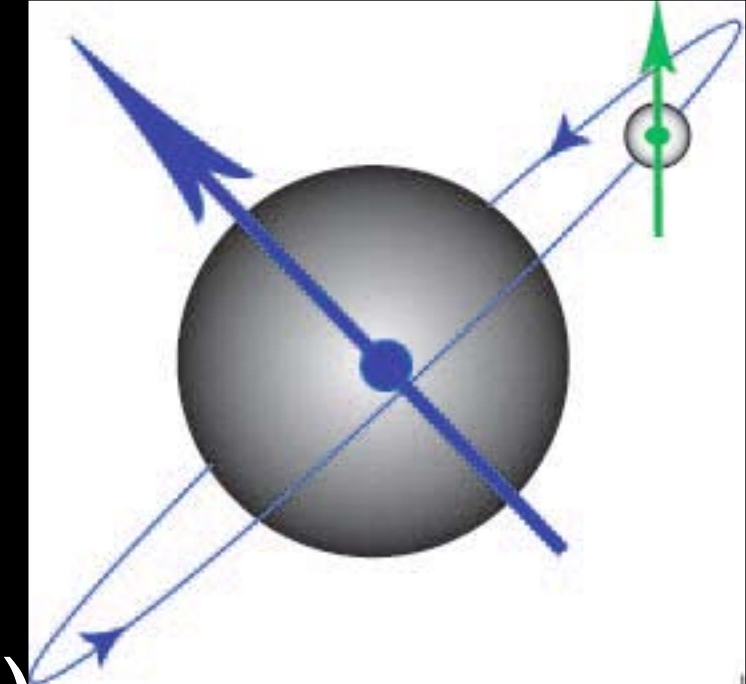


FIG. 1. The specific heat of the two YIG samples. The points for sample 1 give the results obtained in one experiment. The points for sample 2 give the results obtained in two separate experiments.



nor BEC



ms (ferromagnetic)

- SO(3)xU(1) → SO(2)
- $G/H = \mathbb{R}P^3$
- 3 broken generators
- 1 NGB with $E \propto p$
- 1 NGB with $E \propto p^2$

$$\psi = \begin{pmatrix} \psi_x \\ \psi_y \\ \psi_z \end{pmatrix} = \begin{pmatrix} R_x & I_x \\ R_y & I_y \\ R_z & I_z \end{pmatrix}$$

$$\langle \psi \rangle = v \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} = v \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

VOLUME 88, NUMBER 11 PHYSICAL REVIEW LETTERS 18 MARCH 2002

Spontaneous Symmetry Breaking with Abnormal Number of Nambu-Goldstone Bosons and Kaon Condensate

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PHYSICAL REVIEW D **70**, 014006 (2004)

Abnormal number of Nambu-Goldstone bosons in the color-asymmetric dense color superconducting phase of a Nambu−Jona-Lasinio−type model

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PHYSICAL REVIEW A **74**, 033604 (2006)

Superfluidity in a three-flavor Fermi gas with SU(3) symmetry

Lianyi He, Meng Jin, and Pengfei Zhuang
Physics Department, Tsinghua University, Beijing 100084, China

(Received 26 April 2006; published 8 September 2006)

We investigate the superfluidity and the associated Nambu-Goldstone modes in a three-flavor atomic Fermi gas with SU(3) global symmetry. The *s*-wave pairing occurs in flavor antitriplet channel due to the Pauli principle, and the superfluid state contains both gapped and gapless fermionic excitations. Corresponding to the spontaneous breaking of the SU(3) symmetry to a SU(2) symmetry with five broken generators, there are only three Nambu-Goldstone modes, one is with linear dispersion law and two are with quadratic dispersion law. The other two expected Nambu-Goldstone modes become massive with a mass gap of the order of the fermion energy gap in a wide coupling range. The abnormal number of Nambu-Goldstone modes, the quadratic dispersion law, and the mass gap have significant effect on the low-temperature thermodynamics of the matter.

DOI: 10.1103/PhysRevA.74.033604 PACS number(s): 03.75.Ss, 05.30.Fk, 74.20.Fg, 34.90.+q

Spontaneous Breaking of Lie and Current Algebras

Yoichiro Nambu[1]

Received December 26, 2002; accepted January 29, 2003

The anomalous properties of Nambu–Goldstone bosons, found by Miransky and others in the symmetry breaking induced by a chemical potential, are attributed to the SSB of Lie and current algebras. Ferromagnetism, antiferromagnetism, and their relativistic analogs are discussed as examples.[2]

KEY WORDS: Symmetry breaking; Nambu–Goldstone boson; color superconductivity; chemical potential; ferromganetism; Lorentz symmetry; current algebra.

1. INTRODUCTION AND SUMMARY

In general the number of the Nambu–Goldstone (NG) bosons associated with a spontaneous symmetry breaking (SSB) $G \to H$ is equal to the number of symmetry generators Q_i in the coset G/H. In the absence of a gauge field, their energy ω goes as a power k^γ of wave number. In a relativistic theory, $\gamma = 1$ necessarily unless Lorentz invariance is broken.

There are, however, exceptions to the above "theorem."[(1–5)] Recently

Main points
= simple points

H. Watanabe and HM, arXiv:1203.0609

see also Y. Hidaka, arXiv:1203.1494

Heisenberg models

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

- anti-ferromagnet $H = +J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j$ 2 NGBs

$\langle 0|J_z^0|0\rangle = 0$ $E \propto p$

- ferromagnet $H = -J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j$ 1 NGB

$\langle 0|J_z|0\rangle = -i\langle 0|[J_x, J_y]|0\rangle \neq 0$ $E \propto p^2$

J_x and J_y canonically conjugate to each other *cf.* $[x, p]=i\hbar$
describing a single degree of freedom *together*

General formula

- Define a commutator among broken generators $$\rho_{ab} = \frac{-i}{V}\langle 0|[Q^a, Q^b]|0\rangle$$
- n_B = 1/2 rank ρ counts the number of canonically conjugate pairs (Type-B) generically → $E \propto p^2$
  - each pair describes one d.o.f.
- the remainder $n_A = n_{BG} - 2n_B$
  - stand-alone NGB d.o.f. (Type-A) generically → $E \propto p$

$$n_{NGB} = n_A + n_B = n_{BG} - \frac{1}{2}\text{rank}\rho$$

conjectured by Watanabe and Brauner

Applications

$n_{NGB} = n_{BG} - \frac{1}{2}\text{rank}\rho$

example	coset space	BG	NGB	rank ρ	theorem
anti-ferromagnet	O(3)/O(2)	2	2	0	2=2-0
ferromagnet	O(3)/O(2)	2	1	2	1=2-1
superfluid ^{4}He	U(1)	1	1	0	1=1-0
superfluid ^{3}He B phase	O(3)xO(3)xU(1)/O(2)	4	4	0	4=4-0
(in magnetic field)	O(2)xO(3)xU(1)/O(2)	4	3	2	3=4-1
BEC (F=0)	U(1)	1	1	0	1=1-0
BEC (F=1) polar	O(3)xU(1)/U(1)	3	3	0	3=3-0
BEC (F=1) ferro	O(3)xU(1)/SO(2)	3	2	2	2=3-1
3-comp. Fermi liquid	U(3)/U(2)	5	3	4	3=5-2
neutron star	U(1)	1	1	0	1=1-0
kaon cond. (μ=0)	U(2)/U(1)	3	3	0	3=3-0
kaon cond. (μ≠0)	U(2)/U(1)	3	2	2	2=3-1
crystal	$\mathbb{R}^3/\mathbb{Z}^3$	3	3	0	3=3-0
(in magnetic field)	$\mathbb{R}^3/\mathbb{Z}^3$	3	2	2	2=3-1

Formalism
–Internal Symmetries–

H. Watanabe and HM, arXiv:1203.0609
full paper in preparation

Low-*E* Effective Theory w/ Lorentz-invariance

- consider $\pi^a(x)$ fields: $\mathbb{R}^{3,1} \to G/H$ ("*pions*")
- Write *action* $S=\int d^4x\, L(\pi,\partial\pi)$ which is *G*-invariant
- expand in powers of derivative, keep low orders (often up to the second order)
- Lorentz invariance dictates the action to be $S=\int d^4x\, g_{ab}(\pi)\, \partial_\mu\pi^a\, \partial^\mu\pi^b$
- only data needed is *G*-inv metric on *G*/*H*
- indeed, $n_{\rm NGB}=n_{\rm BG}$

For SO(3)/SO(2)=S^2, $S = F^2 \int d^4x \partial_\mu n^i \partial^\mu n^i$

Low-E Effective Theory w/o Lorentz-invariance

- consider $\pi^a(x)$ fields: $\mathbb{R}^{3,1} \to G/H$ ("*pions*")
- Write *action* $S=\int d^4x\, L(\pi, \partial_t\pi, \partial_x\pi)$ which is G-invariant *up to a surface term*
- expand in powers of derivative, keep low orders (often up to the second order)
- $E=\hbar\omega \propto \partial_t \pi$, $p=\hbar k \propto \partial_x \pi$
- typically up to second powers
- assume translation & rotation inv. of space

non-Lorentz-inv case

$$\mathcal{L}_{\rm eff} = g_{ab}(\pi)\partial_\mu\pi^a\partial^\mu\pi^b$$

$$\mathcal{L}_{\rm eff} = \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

- simple generalization to non-Lorentz invariant case, two "metrics" may differ
- in particular, their relative normalization c_s^2 may not be c^2
- more importantly, there is one additional term possible in general (Leutwyler)

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

spectrum

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

- around the origin $c_a(\pi) = c_a(0) + \frac{1}{2}c_{ab}\pi^b + O(\pi^2)$
- in the subspace where c_{ab} is invertible, $L = p\dot{q} - H$

$$[\pi^a, \pi^b] = -i(c^{-1})^{ab}$$

- the c_a term dominates over g_{ab} term $E \gg E^2$
- broken Noether currents $j^0_a = \frac{\partial\mathcal{L}_{\rm eff}}{\partial\dot{\pi}^b}\delta_a\pi^b = c_{ba}\pi^b$

$$[j^0_a, Q_b] = -ic_{ca}c_{db}(c^{-1})^{cd} = ic_{ab}$$

- Namely, for $\rho_{ab} = \frac{-i}{V}\langle 0|[Q^a, Q^b]|0\rangle$ $c_a\dot{\pi}^a \approx \frac{1}{2}\rho_{ab}\pi^b\dot{\pi}^a$
- when c_a present, $E \propto p^2$, otherwise $E \propto p$!
- π^a, π^b canonically conjugate, describe 1 dof

Bottomline

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot\pi^a + \bar g_{ab}(\pi)\dot\pi^a\dot\pi^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

- SSB leads to gapless excitations (NGBs)
- Lorentz invariance: $n_{\rm NGB}=n_{\rm BG}$, $E=cp$
- w/o Lorentz invariance:
  - Type A: $\rho_{ab}=0$, $E\propto p$
  - Type B: $\rho_{ab}\neq 0$, $E\propto p^2$
    - $n_{\rm NGB}=n_{\rm A}+n_{\rm B}$
    - $n_{\rm BG}=n_{\rm A}+2n_{\rm B}$
- explicit effective Lagrangian $\Longrightarrow$ interactions
- underlying partially symplectic geometry

$$c_a\dot\pi^a \approx \frac{1}{2}\rho_{ab}\pi^b\dot\pi^a$$

$$\rho_{ab} = \frac{-i}{V}\langle 0|[Q^a, Q^b]|0\rangle$$

For SO(3)/SO(2)=S^2, $$\mathcal{L}_{\rm eff} = \frac{n_x\dot n_y - n_y\dot n_x}{1+n_z} - \vec\nabla n_i\vec\nabla n_i$$

geometry

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot\pi^a + \bar g_{ab}(\pi)\dot\pi^a\dot\pi^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

- What is $c_a(\pi)$?
- it defines one-form $c_1=c_a(\pi)\, d\pi^a$ on *G/H*
- *L* must be *G*-invariant up to a surface term

$$\mathcal{L}_{V_i} c_1 = d\chi$$

- its exterior derivative is *G*-invariant

$$\omega_2 = dc_1$$

$$\mathcal{L}_{V_i}\omega_2 = d\mathcal{L}_{V_i}c_1 = d^2\chi = 0$$

- Namely, *G/H* is endowed with a *G*-invariant closed two-form ω_2 (may be degenerate)

Darboux's theorem: $\omega_2 = \sum_i dp_i \wedge dq_i$

presymplectic structure

geometry

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

- What is $c_a(\pi)$?
- it defines one-form $c_1 = c_a(\pi)\, d\pi^a$ on *G/H*
- *L* must be *G*-invariant up to a surface term

$$\mathcal{L}_{V_i} c = i_{V_i} dc + d(i_{V_i} c) = \boxed{de_i} + d(i_{V_i} c) \qquad de_i = i_{V_i}\omega$$

- the Noether current picks up surface term

$$j_i^0 = -\bar{g}_{ab} h_i^a \dot{\pi}^b + e_i$$

- in the ground state = stationary:

$$\langle 0 | j_i^0 | 0 \rangle = e_i(0)$$

- it is "charge density" of the ground state

General Geometry

closed G-inv

$d\, c_1 = \pi^* \omega_2$

$G/H \longleftarrow F$ ⇐ Type A $E \propto p$

$\downarrow \pi$

symplectic homogeneous ω_2 B ⇐ Type B $E \propto p^2$

$$\omega_2 = \frac{1}{2}\rho_{ab} d\pi^a \wedge d\pi^b + O(\pi)^3 \qquad \rho_{ab} = \frac{-i}{V}\langle 0|[Q^a, Q^b]|0\rangle$$

NGBs for generators *a* and *b* are symplectic pairs and describe a single degree of freedom

$$\mathrm{dim} G - \mathrm{dim} H = n_A + 2 n_B$$

projection possible for compact semi-simple

explicit construction

- for compact semi-simple case, we found closed expressions

$$gU = U'h'(\pi', g)$$

$$\omega = U^{-1}dU = \sum T_k \omega^k$$

$$g_{ab}(\pi) = g_{cd}(0)\omega_a^c(\pi)\omega_b^d(\pi)$$

$$\langle 0|j_i^0|0\rangle = e_i(0)$$

$$c = -\omega^k e_k(0)$$

central extension

$$\mathcal{L}_{\text{eff}} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

$$de_i = i_{V_i}\omega$$

$$\mathcal{L}_{V_j} de_j = \mathcal{L}_{V_j} i_{V_i}\omega = i_{[V_j,V_i]}\omega + i_{V_i}\mathcal{L}_{V_j}\omega = f_{ji}{}^k i_{V_k}\omega = f_{ji}{}^k de_k$$

$$\mathcal{L}_{V_j} e_j = f_{ji}{}^k e_k + z_{ji}$$

- If $H^2(\mathfrak{g})\neq 0$, a central extension $z_{ji}\neq 0$ possible
- impossible for semi-simple $\mathfrak{g}$
- possible for multiple U(1)'s, $\mathbb{R}$'s
- important when magnetic field

Examples: n_{BG}=1

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

- spontaneously broken U(1): scalar BEC

$$\mathcal{L} = \frac{1}{2}\dot{\theta}^2 - \frac{1}{2}c_s^2(\vec{\nabla}\theta)^2$$

- the only difference from Lorentz-invariant case is the metric can have different normalization for space and time
- it is the speed of sound

Examples: n_{BG}=2

$$\mathcal{L}_{\text{eff}} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

- $\mathbb{R}^2$: invariant closed two-form is

$$\omega_2 = dx \wedge dy = \frac{i}{2} dz \wedge d\bar{z}$$
$$c_1 = \frac{i}{2} z d\bar{z}$$

- the leading terms are

$$\mathcal{L}_{\text{eff}} = \frac{i}{2} z\dot{\bar{z}} - \frac{1}{2m}\vec{\nabla} z \vec{\nabla}\bar{z}$$

- free non-rel particle with one dof, $E \propto p^2$
- or 2d lattice in B, with one dof, $E \propto p^2$

$[z(x), \bar{z}(y)] = -i\delta(x-y)$ central extension $H^2(\mathbb{R}^2) \neq 0$

Examples: n_{BG}=2

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot\pi^a + \bar g_{ab}(\pi)\dot\pi^a\dot\pi^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

- $\mathbb{R}^2$: invariant closed two-form is

$$\omega_2 = dx\wedge dy = \frac{i}{2}dz\wedge d\bar z$$
$$c_1 = \frac{i}{2}z d\bar z$$

- But if c_1 absent, need to consider 2nd term

$$\mathcal{L}_{\rm eff} = \frac{1}{2}(\dot x^2+\dot y^2) - \frac{1}{2}c_s^2((\vec\nabla x)^2 + (\vec\nabla y)^2)$$

- e.g., 2D lattice with two dof, $E \propto p$

Examples: n_{BG}=2

$$\mathcal{L}_{\text{eff}} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

- S^2: SO(3)-invariant closed two-form is

$$\omega_2 = d(\cos\theta) \wedge d\phi$$

$$dc_1 = \frac{1}{2} d\frac{n_y dn_x - n_x dn_y}{1+n_z} = d[(-1+\cos\theta)d\phi]$$

- the leading terms are

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\frac{n_y\dot{n}_x - n_x\dot{n}_y}{1+n_z} - c_s^2\frac{1}{2}\vec{\nabla}n_i\vec{\nabla}n_i$$

- ferromagnet with one dof, $E \propto p^2$

Examples: n_{BG}=2

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

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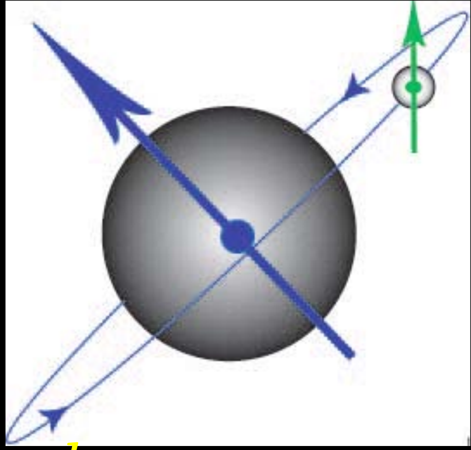
$$\omega_2 = d(\cos\theta)\wedge d\phi$$

$$dc_1 = \frac{1}{2}d\frac{n_y dn_x - n_x dn_y}{1+n_z} = d[(-1+\cos\theta)d\phi]$$

- But if c_1 absent, need to consider 2nd term

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\dot{n}_i\dot{n}_i - c_s^2\frac{1}{2}\vec{\nabla}n_i\vec{\nabla}n_i$$

- anti-ferromagnet with two dof, $E \propto p$



Examples: n_{BG}=3

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot\pi^a + \bar g_{ab}(\pi)\dot\pi^a\dot\pi^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

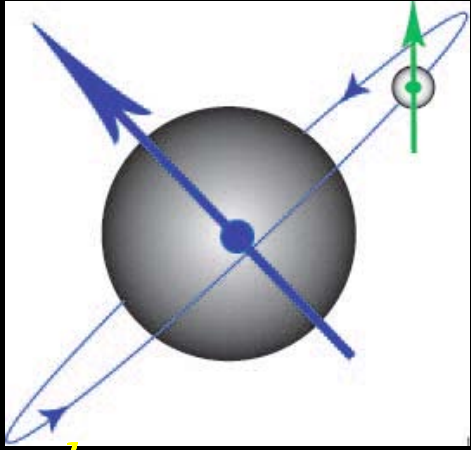
- SO(3)×U(1)/SO(2) = $\mathbb{R}P^3 = S^3/\mathbb{Z}_2$
- spinor BEC ferromagnetic phase:

$$\psi = v\frac{e^{i\theta}}{\sqrt{2}(1+\bar z z)}\begin{pmatrix} 1-z^2 \\ i(1+z^2) \\ 2z \end{pmatrix} \quad \psi^\dagger\psi = v^2$$

$$\psi^* i\dot\psi = v^2\left(-\dot\theta + i\frac{z^*\dot z - \dot z^* z}{1+z^* z}\right)$$

- one dof with $E \propto p^2$, one dof with $E \propto p$

Hopf map $\mathbb{R}P^3 \to S^2$ down to a symplectic homogeneous S^2



Examples: n_{BG}=3

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

- SO(3)×U(1)/SO(2) = $(S^2\times S^1)/\mathbb{Z}_2$
- spinor BEC polar phase:

$$\psi = ve^{i\theta}\vec{n} \quad \vec{n}^2 = 1$$

$$\psi^* i\dot{\psi} = v^2 i\dot{\theta} \approx 0$$

- three dof with $E \propto p$
- vanishing presymplectic structure

Examples: n_{BG}=3

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot\pi^a + \bar g_{ab}(\pi)\dot\pi^a\dot\pi^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

- U(2)/U(1)=S^3 (kaon condensation):

$$\psi = v\frac{e^{i\theta}}{\sqrt{1+\bar z z}}\left(\begin{array}{c} 1 \\ z \end{array}\right) \qquad \psi^\dagger\psi = v^2$$

$$i\psi^\dagger\dot\psi = -\dot\theta + i\frac{1}{2}\frac{\bar z\dot z - \dot{\bar z}z}{1+\bar z z}$$ chemical potential

- the leading terms are

$$\mathcal{L}_{\rm eff} = +i\frac{1}{2}\frac{\bar z\dot z - \dot{\bar z}z}{1+\bar z z} + \left(\dot\theta - i\frac{1}{2}\frac{\bar z\dot z - \dot{\bar z}z}{1+\bar z z}\right)^2 - \left[\left(\vec\nabla\theta - i\frac{1}{2}\frac{\bar z\vec\nabla z - \vec\nabla\bar z z}{1+\bar z z}\right)^2 + 2\frac{\vec\nabla\bar z\vec\nabla z}{(1+\bar z z)^2}\right]$$

- one dof with $E \propto p^2$, one dof with $E \propto p$

Hopf map $S^3 \to S^2$ down to a symplectic homogeneous S^2

General Geometry

U(*N*)/U(*N*–1)=S^{2N-1}

closed *G*-inv

$d\,c = \pi^*\omega_2$

U(1)

G/H ← *F*

Type A

$E \propto p$

π ↓

$\mathbb{C}P^N$ symplectic homogeneous ω_2 *B*

Type B

$E \propto p^2$

$$\omega_2 = \frac{1}{2}\rho_{ab} d\pi^a \wedge d\pi^b + O(\pi)^3 \qquad \rho_{ab} = \frac{-i}{V}\langle 0|[Q^a, Q^b]|0\rangle$$

NGBs for generators *a* and *b* are symplectic pairs and describe a single degree of freedom

$$\dim G - \dim H = n_A + 2n_B$$

Applications

$$n_{NGB} = n_{BG} - \frac{1}{2}\text{rank}\rho$$

example	coset space	BG	NGB	rank ρ	theorem
anti-ferromagnet	O(3)/O(2)	2	2	0	2=2-0
ferromagnet	O(3)/O(2)	2	1	2	1=2-1
superfluid ^{4}He	U(1)	1	1	0	1=1-0
superfluid ^{3}He B phase	O(3)xO(3)xU(1)/O(2)	4	4	0	4=4-0
(in magnetic field)	O(2)xO(3)xU(1)/O(2)	4	3	2	3=4-1
BEC (F=0)	U(1)	1	1	0	1=1-0
BEC (F=1) polar	O(3)xU(1)/U(1)	3	3	0	3=3-0
BEC (F=1) ferro	O(3)xU(1)/SO(2)	3	2	2	2=3-1
3-comp. Fermi liquid	U(3)/U(2)	5	3	4	3=5-2
neutron star	U(1)	1	1	0	1=1-0
kaon cond. (μ=0)	U(2)/U(1)	3	3	0	3=3-0
kaon cond. (μ≠0)	U(2)/U(1)	3	2	2	2=3-1
crystal	$\mathbb{R}^3/\mathbb{Z}^3$	3	3	0	3=3-0
(in magnetic field)	$\mathbb{R}^3/\mathbb{Z}^3$	3	2	2	2=3-1

stability@T=0 in d+1 dim

- Type A:
$$\mathcal{L}_{\text{eff}} = \bar{g}_{ab}\dot{\pi}^a\dot{\pi}^b - g_{ab}\nabla_i\pi^a\nabla\pi^b$$
$$\vec{x}' = a\vec{x},\ t' = at$$
  - scaling $$\pi'^a(a\vec{x}, at) = a^{(1-d)/2}\pi^a(\vec{x}, t)$$
  - interaction $$\nabla_i\pi^a\nabla_i\pi^b\pi^c \sim a^{-(d-1)/2}$$
  - IR free for $d\geq 2$ (d=1 symmetry restored)
- Type B:
$$\mathcal{L}_{\text{eff}} = \rho_{ab}\pi^a\dot{\pi}^b - g_{ab}\nabla_i\pi^a\nabla\pi^b$$
$$\vec{x}' = a\vec{x},\ t' = a^2 t$$
  - scaling $$\pi'^a(a\vec{x}, a^2 t) = a^{-d/2}\pi^a(\vec{x}, t)$$
  - interaction
  - IR free for $d\geq 1$ $$\nabla_i\pi^a\nabla_i\pi^b\pi^c \sim a^{-d/2}$$

à la Hořava-Lifshitz

Previously Known Theorems

Nielsen-Chadha theorem

$$\mathcal{L}_{\rm eff} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

- Type-I if $E \propto p^{2n+1}$
- Type-II if $E \propto p^{2n}$
- Proved $n_{I}+2n_{II} \geq n_{BG}$
- only an inequality, a weak statement
- follows from our result $n_A+2n_B=n_{BG}$ because Type-A (B) is generically Type-I (II)
- but not the same if $O(\nabla^2)$ term absent and L starts with $O(\nabla^4)$, then Type-A but Type-II

Schäfer et al theorem

$$\mathcal{L}_{\text{eff}} = c_a(\pi)\dot{\pi}^a + \bar{g}_{ab}(\pi)\dot{\pi}^a\dot{\pi}^b - g_{ab}(\pi)\nabla_i\pi^a\nabla_i\pi^b$$

$$c_a\dot{\pi}^a \approx \frac{1}{2}\rho_{ab}\pi^b\dot{\pi}^a$$

- If $\rho_{ab} = -i \langle 0| [Q_a, Q_b] |0\rangle / V = 0$
- then no Type-B
- $n_{NGB} = n_{BG} = n_A$

$$n_{NGB} = n_{BG} - \frac{1}{2}\text{rank}\rho$$

conjectured by Watanabe and Brauner

no-go case

- Not every NGBs can be paired as Type-B
- SU(3)/U(1)^2: Kähler and symplectic

Type-A	Type-B	$n_A+2n_B=6$
6	0	6
4	1	6
2	2	6
0	3	6

redundancies

H. Watanabe and HM, arXiv:1203.0609
another one in preparation w/ T. Brauner

spacetime symmetries

- so far all discussions are internal symmetries
- but there are situations when n_{NGB} is further reduced for spacetime symmetries
- spontaneously broken scale and conformal symmetries lead to only one NGB (dilaton) (Salam-Strathdee)
- crystal breaks both translations (P_i) and rotations (J_i), but only phonons for P_i

Noether constraints

- They can be understood as a consequence of *Noether constraints* $\int d^d x \sum_a c_a(x) j_a^0(x)|0\rangle = 0$
- For broken symmetries, we have $\langle \pi_b | j_a^0(x) | 0\rangle \neq 0$
- then they are linearly *redundant*

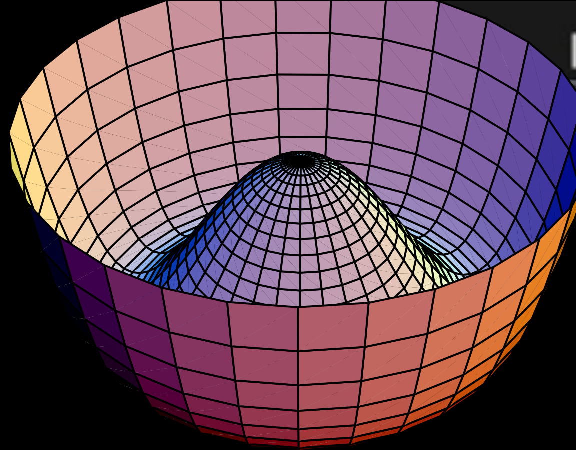
$$0 = \sum_b |\pi_b\rangle\langle\pi_b| \int d^d x \sum_a c_a(x) j_a^0(x)|0\rangle$$

$$= \sum_b |\pi_b\rangle \int d^d x c_a(x) \langle \pi_b | j_b^0(x)|0\rangle$$

Examples

- crystal: translations and rotations are both spontaneously broken
- they are both generated by the energy-momentum tensor $R^{0i} = \epsilon_{ijk} x^j T^{0k}$
- would-be NGBs for rotations are the same excitations as those for translations (phonons)

Examples



- Ginzburg-Landau theory

$$V = -\mu\psi^*\psi + \lambda(\psi^*\psi)^2$$

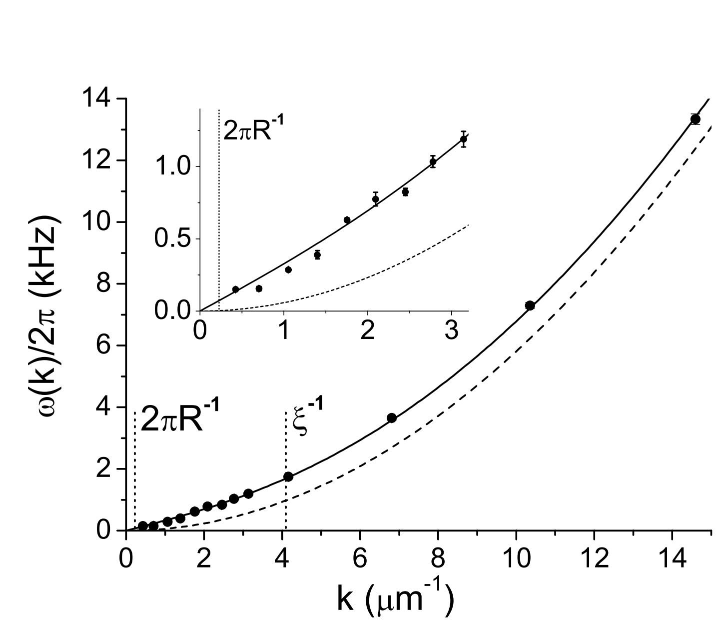
- G=U(1), H=0
- ^{4}He superfluid
- scalar BEC $\langle 0|\psi|0\rangle \neq 0$
- U(1) $\psi(\vec{x},t) \to e^{i\theta}\psi(\vec{x},t)$
- Galilean boost

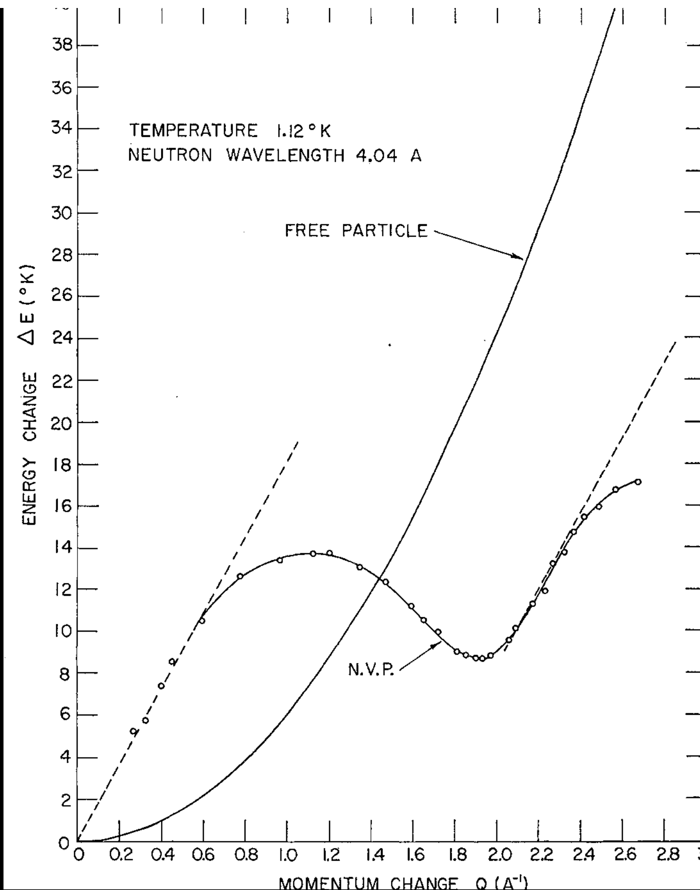
$$\psi(\vec{x},t) \to e^{i(m\vec{x}\cdot\vec{x}-\frac{1}{2}m\vec{v}^2 t)}\psi(\vec{x}-\vec{v}t,t)$$

- both broken n_{BG}=1+3=4

$$B^{i\mu} = tT^{i\mu} - mx^i j^\mu$$

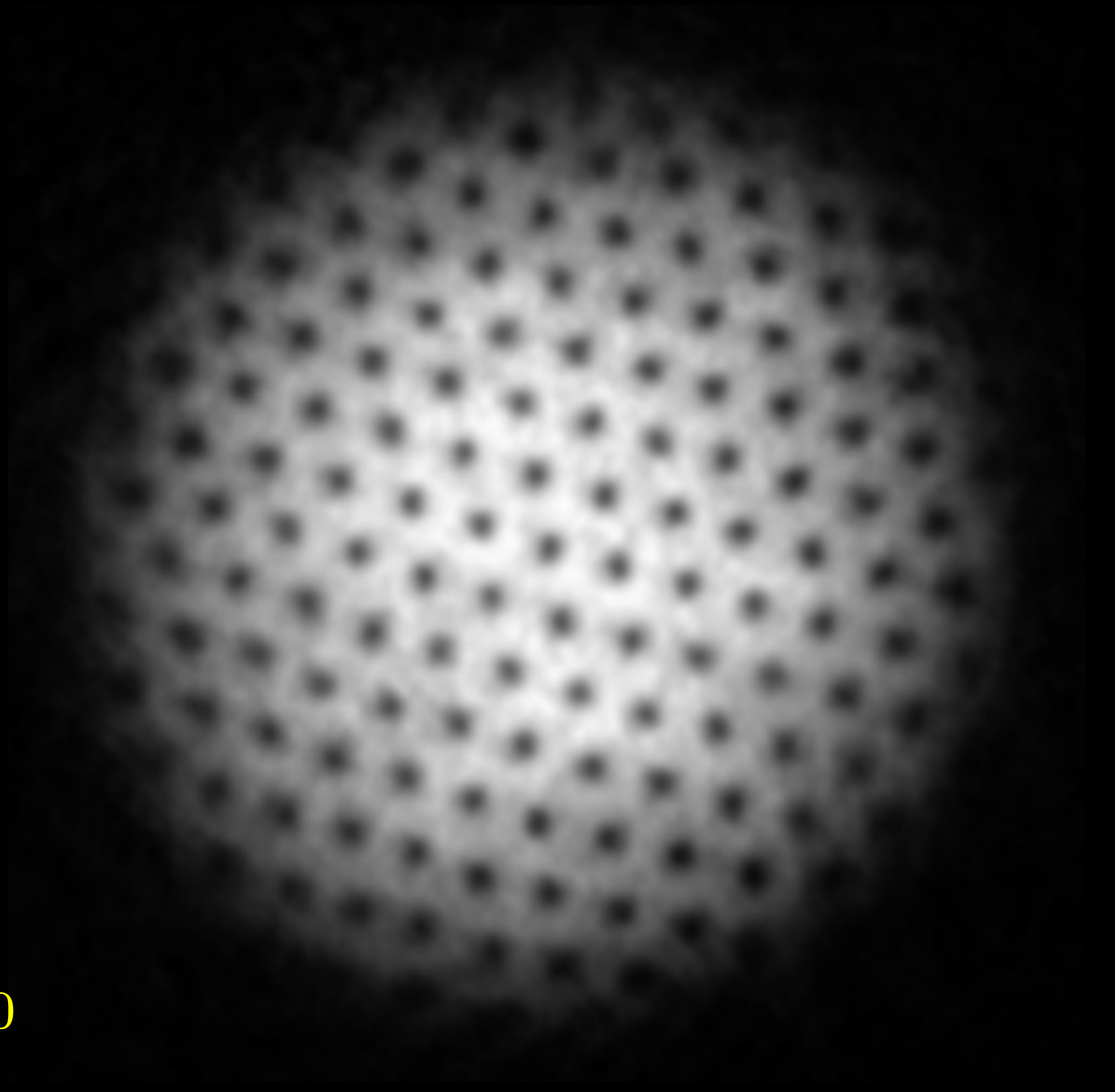
⇒ no separate NGBs for Galilean boosts





vortex lattice

- rotate a (2d) BEC
- vortices form a triangular lattice
- broken: U(1), $P_{x,y}$, J_z
- only one Type-A NGB with $E \propto p^2$
- called Tkachenko mode



$$T^{0i} = mj^i - 2m\Omega\epsilon^{ij}x^j j^0$$

we have a precise effective Lagrangian for this

relation to inverse Higgs mechanism

- old idea called "*inverse Higgs mechanism*" is used to eliminate spurious NGBs in case of spacetime symmetries E.A. Ivanov and V.I. Ogievetskii, 1975
- not much discussed in cases without Lorentz invariance
- not applicable when translation is broken
- recently more papers
  - Endlich, Nicolis, Penco, arXiv:1310.2272
  - Hayata, Hidaka, arXiv:1312.0008

massive NGB

H. Watanabe, T. Brauner, and HM, arXiv:1303.1527

Nicolis-Piazza

- normally, we can say few things about gapped modes based on symmetries alone (cf. BPS)
- They pointed out in Lorentz-invariant systems and broken symmetries, some gaps can be predicted *exactly* with group theory

$$\tilde{H} = H - \mu Q$$

$$[Q, E_{\pm\alpha}] = \pm\alpha E_{\pm\alpha}$$

$$\tilde{H}(E_\alpha|0\rangle) = \mu\alpha(E_\alpha|0\rangle)$$

but not for the conjugate generator

massive NGB

- It turns out the system does not need to be Lorentz invariant, nor *Q* broken
- quite general result applicable in many systems

$$n_{mNGB} = \frac{1}{2}(\mathrm{rank}\rho - \mathrm{rank}\tilde{\rho})$$

$$\rho_{ab} = \frac{-i}{V}\langle 0|[Q_a, Q_b]|0\rangle \qquad [Q_a, H] = 0$$

$$\tilde{\rho}_{ab} = \frac{-i}{V}\langle 0|[\tilde{Q}_a, \tilde{Q}_b]|0\rangle \qquad [\tilde{Q}_a, \tilde{H}] = [\tilde{Q}_a, H - \mu Q] = 0$$

$$\tilde{H}(E_\alpha|0\rangle) = \mu\alpha(E_\alpha|0\rangle)$$

Examples

- ferromagnet and anti-ferromagnet in a constant magnetic field
- relativistic BECs, kaon condensation
- QCD with chemical potential for isospin
- many examples previously known based on approximation methods, now are *exact*

Conclusion

- age-old subject, yet still a lot to learn!
- for internal symmetries, precise counting rule and dispersion relation of NGBs finally known
- underlying geometry: *presymplectic structure*
- redundancies make sense for spacetime symmetries
- exact predictions on *massive NGBs* à la BPS

IL NUOVO CIMENTO   VOL. XIX, N. 1   1° Gennaio 1961

Field Theories with «Superconductor» Solutions.

J. GOLDSTONE

CERN - Geneva

(ricevuto l'8 Settembre 1960)

Summary. — The conditions for the existence of non-perturbative type « superconductor » solutions of field theories are examined. A non-covariant canonical transformation method is used to find such solutions for a theory of a fermion interacting with a pseudoscalar boson. A covariant renormalisable method using Feynman integrals is then given. A « superconductor » solution is found whenever in the normal perturbative-type solution the boson mass squared is negative and the coupling constants satisfy certain inequalities. The symmetry properties of such solutions are examined with the aid of a simple model of self-interacting boson fields. The solutions have lower symmetry than the Lagrangian, and contain mass zero bosons.

Sanke

ノーベル賞南部理論 自然界に拡張

東大研究者ら発表

米シカゴ大名誉教授の南部陽一郎博士が1961年に提唱し、ノーベル物理学賞の受賞対象となった「対称性の自発的な破れ」の理論を一般化した統一理論を、東京大・カブリ数物連携宇宙研究機構の村山斉機構長らが8日、米物理学会誌（電子版）に発表した。

南部博士の理論は、絶対零度で真空という条件下での素粒子を想定したもので、温度や密度のある普通の物質で起こる現象では成り立たないケースが多くあることが知られていた。

村山さんは米カリフォルニア大バークレー校の大学院生、渡辺悠樹さんと共同で南部理論の

拡張に取り組
発的な破れ」
だした。

この成果は
野に波及し、
も期待でき

対称性の自
をさせても
性」が、外か
と。南部理論
れに対応する
が、物質に

例えば左右
性が破れると、南部理論では2つの波が現れるはずだが、このような破れが起こる磁石では波は1つしか現れない。村山さんらは2つの対称性の破れが、一緒に1つの波を生み出す場合があることを理論的に導いた。

Yomiuri

「対称性の破れ」問題点解決

米の大学院生ら 条件問わぬ新公式作成

南部陽一郎博士がノーベル物理学賞を受賞する業績となった「対称性の自発的な破れ」理論の問題点を、米カリフォルニア大バークレー校大学院生の渡辺悠樹さん(25)らが解決した。現代数学の手法を駆使して新しい公式を作り、南部理論がこれまで成り立たなかった条件にも適用できるようにした。8日付の米物理学会誌「フィジカル・レビュー・レターズ」電子版に掲載され、1960年代の理論提唱から半世紀に及んだ問題に終止符が打たれる。

磁石の中の小さな粒子は、高温の時はばらばらな向きで激しく運動している。温度が下がると一部の粒子の向きがそろい、周りの粒子も次々と同じ向きになる。このように、本来はどちらを向いても構わないはずの物質が一定の向きにそろう現象が、「対称性の自発的な破れ」。南部博士は、最初に向きのそろった部分から、波のように他の部分にも伝わっていくとえ、素粒子から宇宙まで様々な領域で「自発的破れ」の現象を説明できる式を作った。

ただ、絶対温度が0度で真空という条件を想定した式のため、説明できない現象もあった。渡辺さんらは複数の波が、条件によっては絡み合って一つの波になることを突き止めた。そて、波が絡み合うケースを考慮し、温度や密度によらずに成立する公式を作ることに成功した。

共同研究者の村山斉・東京大学カブリ数物連携宇宙研究機構長は「宇宙の成り立ちや新しい物性の理解が進むのではないか」と話している。

Mainichi

ノーベル賞受賞南部理論 例外なく証明成功

東大チーム

08年にノーベル物理学賞を受賞した南部陽一郎博士が60年代に提唱した「対称性の自発的な破れ」理論で、カリフォルニア大と東京大の研究チームがこれを発展させ、従来説明し切れなかった範囲まで例外なく証明できる理論を導き出した。

物理学界の50年来の謎を解き明かす成果として、8日付の米物理学会誌「フィジカルレビューレターズ」に掲載された。

例えば、洗濯物を干す時、シャツをどちら向きから干しても構わない（対称性）が、最初にたまたま右向きに干すと、結果的にすべてのシャツが右向きに並ぶことがある。南部博士の理論は、この意図せず左右の向きが決まることを「対称性の自発的な破れ」としている。

博士は、こうした現象を物質の最小単位の素粒子の世界に当てはめ、ノーベル賞受賞につながった。

ただ、絶対零度の真空を前提としており、密度や温度がある生まれたての宇宙や、磁石の中など身の回りで説明できないものもあることが分かってい
た。

カリフォルニア大大
学院生の渡辺悠樹さ
(25)は、南部博士の
理論を数学的に発展さ
せ、「対称性の自発的な破れ」をすべて説明できる理論を導き出した。東京大の村山斉特任教授（素粒子物理学）は「現状では成果をどの分野に応用できるか分からない」と話すが、宇宙の進化過程の解明など幅広い分野に応用される可能性があるという。【鳥井真平】

Asahi

南部理論を改良する公式

東大特任教授ら 米学会誌発表へ

素粒子物理学でノーベル賞を受賞した南部陽一郎さんの理論を改良し、より幅広い世界の現象を説明できる公式を東京大の村山斉特任教授らが作った。米物理学会誌で9日、発表する。

物質の内部などで電子などがきれいに並んでいる状態を「対称性が自発的に破れている」状態と言い、南部さんの理論では、このときその物質に固有の「破れの数」が存在する。破れの数は、その状態で生じる特殊な「波」の数と同じだと指摘した。

だが、この理論で説明できるのは、「物質の温度が絶対零度で密度もゼロ」という状態に限られる。温度や密度がある現実世界の状態では、破れの数と波の数が合わないことが多く、50年近い懸案になっていた。

村山さんとカリフォルニア大バークリー校大学院生の渡辺悠樹さんは、数学の理論を組み合わせ、現実世界で、破れの数から波の数を割り出す公式を編み出した。

（高山裕喜）

pathology

- In the presence of central extension, there are examples where the presymplectic structure cannot be projected down to a symplectic homogeneous space
- T^3 with $\omega = d\theta^1 \wedge (d\theta^2 + r d\theta^3)$
- if r irrational, the projection would be dense and ill-defined
- I consider such a case *pathological*