129B HW # 1 (due Jan 30)

The amplitude of muon decay is proportional to the Fermi constant G_F which has a dimension of inverse mass squared in the natural unit.

1. Argue that the decay rate must be proportional to $G_F^2 m_\mu^5$ using the dimensional analysis. (We neglect the electron mass in muon decay, which gives an error only of the order of $m_e^2/m_\mu^2 < 3 \times 10^{-5}$.)

2. Calculate $\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)$, $\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$, $\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)$ from the data given in the Particle Data Group booklet, including the error bars for the latter two.

3. Test the hypothesis of charged current universality, by comparing the above three quantities. Namely, calculate the ratios $G_F^{\tau e}/G_F^{\mu e}$ and $G_F^{\tau\mu}/G_F^{\mu e}$ and show that they are both consistent with 1.

optional:

(a) The three-body phase space can be simplified to

$$\int d\Phi_3 = \frac{1}{32\pi^3} \int dE_{\nu_e} \int dE_e \tag{1}$$

in the muon rest frame. The integration is done in a triangular region, $0 \le E_{\nu_e} \le m_\mu/2$, $0 \le E_e \le m_\mu/2$, $m_\mu/2 \le E_{\nu_e} + E_e \le m_\mu$. On the other hand, the squared matrix element summed over helicities is given by

$$\sum_{\text{helicities}} |\mathcal{M}|^2 = 128 G_F^2 (p_{\nu_\mu} \cdot p_e)(p_{\nu_e} \cdot P), \tag{2}$$

where P is the muon four-momentum. Using Fermi's golden rule and performing the phase space integral, show that

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \frac{1}{192\pi^3} G_F^2 m_\mu^5. \tag{3}$$

Calculate G_F from the observed muon lifetime and compare it to the value given on page 3 in the booklet. (Hint: $2(p_{\nu_\mu} \cdot p_e) = (p_{\nu_\mu} + p_e)^2 = (P - p_{\nu_e})^2 = m_\mu^2 - 2 m_\mu E_{\nu_e}$.)

(b) Prove Eq. (1) after three angular integrations.